MODULE-01

INTRODUCTION TO STRUCTURAL ANALYSIS AND ANALYSIS OF PLANE TRUSSES

Structure

Structure is an assemblage of a number of components like slabs, beams, columns, walls, foundations and so on, which remains in equilibrium. It has to satisfy the fundamental criteria of strength, stiffness, economy, durability and compatibility, for its existence. It is generally classified into two categories as Determinate structures and Indeterminate structures or Redundant structures.

Analysis

Determinate structures are analyzed just by the use of basic equilibrium equations. By this analysis, the unknown reactions are found for the further determination of stresses. Redundant or indeterminate structures are not capable of being analyzed by mere use of basic equilibrium equations. Along with the basic equilibrium equations, some extra conditions are required to be used like compatibility conditions of deformations etc. to get the unknown reactions for drawing bending moment and shear force diagrams.

Stable and Unstable Structures

A stable structure is one that will not collapse when disturbed. Stability may also be defined as "The power to recover equilibrium ". In general, there are many ways that a structure may become unstable, including buckling of compression members, yielding/rupture of members, or nonlinear geometric effects like P-Delta; however, for linear structural analysis, the main concern is instability caused by insufficient reaction points or poor layout of structural members. An internally stable structure is one that would maintain its shape if all the reactions supports were removed. A structure that is internally unstable may still be stable if it has sufficient external support reactions.

An unstable structure generally cannot be analyzed. Therefore, it is useful to know if a structure is stable or unstable before a structural analysis is conducted. There are four main ways that a structure may be geometrically unstable. These apply only to linear geometric stability and not to instability caused by buckling, member yielding or nonlinear geometry.

Support Types/Components

In the equations above, *r* is equal to the total number of reaction components as follows:

1. Roller (*r)* = 1
2. Pin (*r)* = 2
3. Fixed (*r)* = 3

For multiple reaction points, (*r)* is the sum of all the components for all the reaction points in the structure.

Equilibrium

The objectives of any structural analysis is the determination of reactions at supports and internal actions (bending moments, shearing forces, etc.). A correct solution for any of these quantities must satisfy the equations of equilibrium:

$\sum Fx=0$

$\sum Fy=0$

$\sum M=0$

Statically Determinate Structure

When the equations of equilibrium are sufficient to determine the forces and stresses in a structure, we say that this Structure is **statically determinate.**

Example-Simply supported beam, cantilever beam, three hinged arc

Statically Indeterminate Structure

A structure is termed as statically indeterminate, if it cannot be analyzed from equations of equilibrium.

Example-Fixed beam, continuous beam, two hinged arch

Redundancy and Degree of Indeterminacy

Indeterminate structures effectively have more unknowns than can be solved using the three equilibrium equations (or six equilibrium equations in 3D). The extra unknowns are called **redundants.**

The degree of indeterminacy is equal to the number of redundant. An indeterminate structure with 2 redundants may be said to be statically indeterminate to the **second degree.**

Difference between Determinate and Indeterminate Structures

1. Determinate Structures Equilibrium conditions are fully adequate to analyze the structure; **while** indeterminate Structures Conditions of equilibrium are not adequate to fully analyze the structure.
2. Determinate Structures The bending moment or shear force at any section is independent of the cross-section or moment of inertia; **while** Indeterminate Structures The bending moment or shear force at any section depends upon the cross-section or moment of inertia.
3. Determinate Structures Temperature variations do not cause stresses; **while** Indeterminate Structures Temperature variations cause stresses.
4. Determinate Structures No stresses are caused due to lack of fit; **while** Indeterminate Structures Stresses are caused due to lack of fit.
5. Determinate Structures Extra conditions like compatibility of displacements are not required to analyze the structure; **while** Indeterminate Structures Extra conditions like compatibility of displacements are required to analyze the structure along with the equilibrium equations.
6. Determinate Structures Bending moment or shear force at any section is independent of the material property of the structure; **while** Indeterminate Structures Bending moment or shear force at any section depends upon the material property.

Linear And Non-Linear Analysis of System

Linear Analysis

- Material is in elastic state obeys Hooks law.
- The behavior of linear structure can be analyzed using linear number of equation.
- Linear system has to obey the principle of super position.

Non-linear Analysis

- The non-linear Analysis allows non-linear stress-strain relationships.
- The stress in the material vary with the amount of deformation.
- Non-linearity can be caused by change in geometry or material behavior.
- It is accurate but not easy as linear analysis.

Types of Non-linearity

1. Material Non-linearity

This is caused due to stress strain relationship.

2. Geometric Non-linearity

This is caused due changes in geometry stiffness changes due to load application.

Degree of freedom(DOF)

The number of displacement allowed at the joints of the structure is called as degree of freedom.
Free End-DOF=3
Roller or simple support-DOF=2
Fixed support-DOF=0

Forms of Structure or Structural Form

The assembly of different components or elements is normally referred as structure. In building consisting of walls, floor, roofs, beams, columns and foundation.
Structural forms can be explained based on 1, 2, and 3 dimensional structural system.

1-Dimensional Structure

The elements are arranged along one axis. may be x or y or z axis.
Example: Beam elements

2-Dimensional structure

If the element are arranged along two axis may be x and y or y and z or z and x axis are called as two-dimensional structure.

Example: Slab elements, plate elements

3-Dimensional structure

If the elements are arranged along three axis may be x-y-z or y-z-x or z-x-y is called as 3-dimesional structure.

Example: Building frames, space frames, steel truss, space truss

Degree of static Indeterminacy[Ds]

The number of additional equation necessary to solve the problem is called static indeterminacy.

For Beams

Ds=r-3-c

Where, r= The number of reaction

3= Number of equilibrium condition

C= Number of hinge

For Plane frames

Ds=(3m+r)-3j

Where, m=number of members

J=number of joints

For plane Truss

Ds= (m+r)-2j

Degree of kinematic indeterminacy [Dk]

The number of equilibrium condition are required to find the displacement component of all joints of the structure [DOF]

For beams and plane Frames

Dk=3j-r+c

For Truss

Dk=2j-r

ANALYSIS OF TRUSSES

Important Definition:

1. Deflection:
It is a temporary phenomenon of displacement due to movement action.

2. Deformation:
It is a permanent displacement due to load action.

3. Displacement:
It can be defined as the movement of individual points on a structural system due to various external load.

4. Free body Diagram:
Part of the structure with external load applied to the members and internal forces developed in the members.

5. Truss:
A truss is a structure consisting of members or element that takes only axial forces (tension or compression)

CLASSIFICATION OF TRUSSES

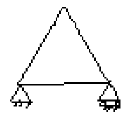
Classification based on structure:

1. Plane or Planar truss:
A member lies in one plane or two dimensional plane.

2. Space truss:
It consists of members jointed together at their ends to form 3D structure.

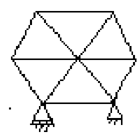
Classification based on co-planar trusses

1. Simple trusses:
It is a planar truss which begins with triangular element and can be expanded by adding two members and joints.



2. Compound truss:
This truss is formed by connecting 2or more simple trusses together. they are often used for large span.

3. Complex truss
This is a truss that can't be classified as being either simple or compound.



Classification based on requirement

1. Bridge truss
A bridge truss is a truss whose load bearing superstructure is composed of a truss and structure of the connected element forms triangular unit.

2. Roof truss
These are structural components of houses and commercial buildings.

Classification based on stability of truss

1.Perfect truss or stable truss

A truss which does not change its shape and maintains equilibrium under the action of load.

$$M=2j-r$$

2.Imperfect truss

Structure is made up of members more or less than the minimum member necessary to keep it in equilibrium condition. When loading then it is called as unstable truss.

M # 2j-r

Types of imperfect truss

Deficient truss: Structure is made up of members less than the joints and reaction components. M<2j-r

Redundant truss: structure is made up of members more than the joints and reaction components. M>2j-r

Components of truss

Chord: These are the members which form the outline of the truss.

Diagonals: These are inclined members inside the truss.

Verticals: These are the vertical members in the truss.

NOTE: If the member of the truss comes under compression it is called as struct and if it comes tension called tie.

ASSUMPTION MADE IN THE ANALYSIS OF TRUSS

1. Members are straight and there is no eccentricity.
2. Self weight of member is neglected.
3. Loads and reaction are only transfer at joints.
4. Joints are frictionless pins.
5. Young's modulus is same throughout.
6. The truss is perfect (m = 2j-r)
7. The members are subjected to axial forces only.
8. Bending moment and shear force are neglected.
9. Obeys hook's law.

Procedure for the analysis of truss (method of joints)

Step 1. Find the reaction at supporting pins using the force and the moment equations

Step 2. Start with a pin, most preferably roller pin,wher there are 2 or less than two unknowns.

Step 3. Proceed in a similar way and try to find out force in different members one by one.

Step 4. Take care of while labelling forces on the members. Indicate compression and tension clearly. Step 5. Finally produce a completely labelled diagram.

Step 6.Try to identify the zero force members. It makes the problem simple.

Problems on Calculation of Static and Kinematic Indeterminacy

Beams

1.

Static Indeterminacy $= D_S = r - 3 - c$

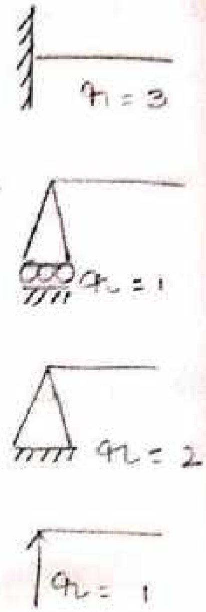
$= 6 - 3 - 0$

$D_S = 3$

Kinematic Indeterminacy $= D_K = 3j - r + c$

$= (3 \times 2) - 6 + 0$

$D_K = 0$



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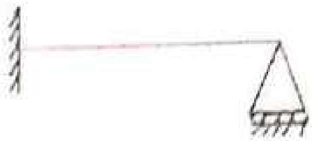


Static Indeterminacy $= D_S = r - 3 - c$

$= 4 - 3 - 0$

$= 1$

Kinematic Indeterminacy $= D_K = 3j - r + c$

$= 3(2) - 4 + 0$

$= 6 - 4$

$D_K = 2$

3

Static Indeterminacy $= D_S = r - 3 - c$

$= 4 - 3$

$= 1$

$D_K = 3j - r + c$

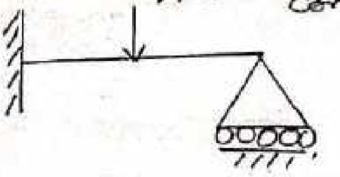
$= 3(2) - 4 + 0$

$= 6 - 4$

$= 2$

→ Donot consider as a reaction

4.



$D_S = r - 3 - c$

$= 4 - 3 - 0$

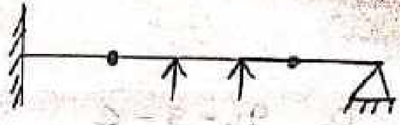
$\boxed{D_S = 1}$

$D_K = 3j - r + c$

$= 3 \times 2 - 4$

$\boxed{D_k = 2}$.

8.



$D_S = r - 3 - c$

$= 7 - 3 - 2$.

$= 2$

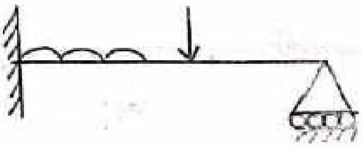
$D_K = 3j - r + c$

$= 3[4] - 7 + 2$

$= 12 - 7 + 2$

$= 7$

5.



$D_S = r - 3 + c$

$= 4 - 3 - 0$

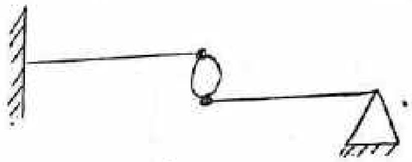
$= 1$

$D_K = 3j - r + c$

$= (3 \times 2) - 4$

$= 2$.

9.



$D_S = r - 3 - c$

$= 5 - 3 - 2$

$= 0$

$D_K = 3j - r + c$

$= 3 \times 4 - 5 + 2$

$= 12 - 5 + 2$

$= \underline{\underline{9}}$

6.

$D_S = r - 3 - C$

$= 6 - 3 - 1$

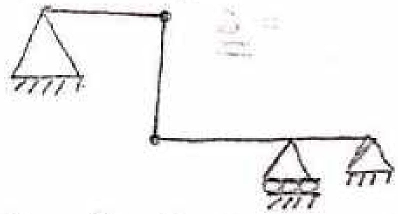
$= 2$.

$D_K = 3j - r + C$

$= 3(2) - 6 + 1$

$= 1$

10.



$D_S = r - 3 - c$

$= 5 - 3 - 2$

$= 0$

$D_K = 3j - r + C$

$= 3[5) - 5 + 2$

$= 15 - 5 + 2$

$= 10 + 2$

$= \underline{\underline{12}}$

7.

$D_S = r - 3 + C$

$= 3 - 3 + 1$

$= -1$

$D_K = 3j - r + C$

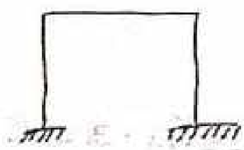
$= 3 \times 2 - 3 + 1$

$D_S = r - 3 - c$
$= 4 - 3 - 0$

$D_K = 3j - r + c$
$= 3(3) - 4 + 0$
$= \underline{\underline{5}}$.

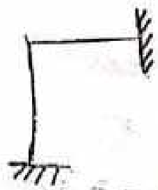
Problems on Frames

1.



$D_S = (3m + r) - 3j$
$= 3 \times 3 + 6 - 3 \times 4$
$= 9 + 6 - 12$
$= 3$

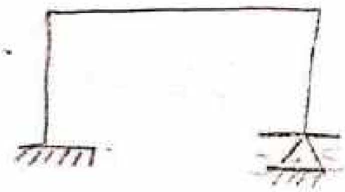
$D_K = 3j - r + c$
$= 3(4) - 6 + 0$
$= \underline{\underline{6}}$.



$D_S = 3m + r - 3j$
$= 3(2) + 6 - 3(3)$
$= 6 + 6 - 9$
$= 3$

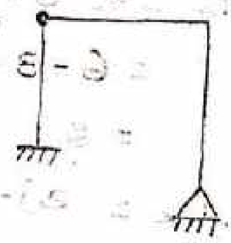
$D_K = 3j - r + c$
$= 3(3) - 6 + 0$
$= 9 - 6$
$= 3$

2.



$D_S = (3m + r) - 3j$
$= (3[3] + 5 - 3[4])$
$= 9 + 5 - 12$
$= 2$

$D_k = 3j - r + c$
$= 3(4) - 5 + 0$
$= 12 - 5$
$= 7$



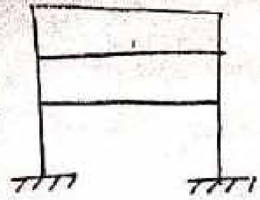
$D_S = 3m + r - 3j - c$
$= 3(3] + 5 - 3(4) - 1$
$= 9 + 5 - 12 - 1$
$= 2 - 1 = 1$

$D_K = 3j - r + c$
$= 3[4) - (5) + 1$
$= 12 - 5 + 1$
$= 8$

$(-1) = 3(3) + 5 - 3(x)$
$(-1) = 9 + 5 - 3x$
$3x = 14 + 1$
$x = 15/3$
$x = 5$

$8 = 3$

5.



$D_s = (3m + r) - 3j$

$= 3\times 5 + 6 - 3(8)$

$= 15 + 6 - 24$

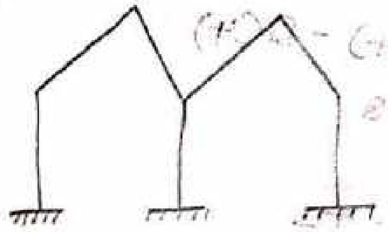
$= 3$

$D_k = 3j - r + c$

$= 3\times 8 - 6$

$= 18$

6.



$D_s = (3m + r) - 3j$

$= 3\times 7 + 9 - 3(8)$

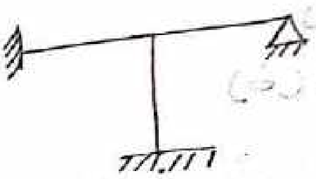
$= 6$

$D_K = 3j - r + c$

$= 3(8) - 9$

$= 15$

7.



$D_s = 3m + r - 3j$

$= 3\times 3 + 8 - 3(4)$

$= 9 + 8 - 12$

$= 5$

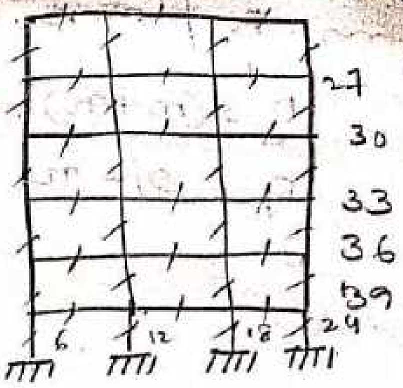
$D_k = 3j - r + c$

$= 3(4) - 8$

$= 12 - 8$

$= 4$

8.



$m = 42$

$D_s = (3m + r) - 3j$

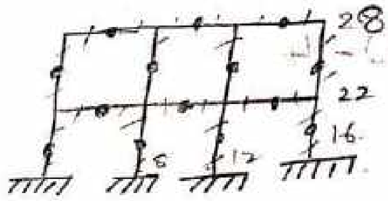
$= (3(42) + 12 - 3(28))$

$= 54$

$D_K = 3j - r + c$

$= 3(28) - 12 + 0$

$= 72.$



$D_s = 3m + r - 3j - c$

$= 3(14) + 12 - 3(12) - 14$

$= 4$

$D_k = 3j - r + c$

$= 3(12) - 12 + 14$

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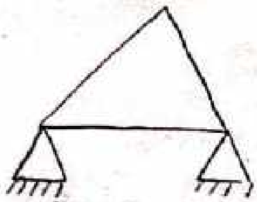
Problems on plane Trusses

$D_S = (m+r) - 2j$

$D_k = 2j - r$

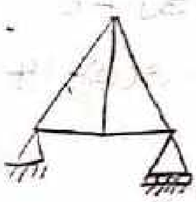
1.



$D_S = (m+r) - 2j$
$= (3+4) - 2(3)$
$= 7-6$
$= \underline{\underline{1}}$

$D_k = 2j - r$
$= 2(3) - 4$
$= \underline{\underline{2}}$

2.

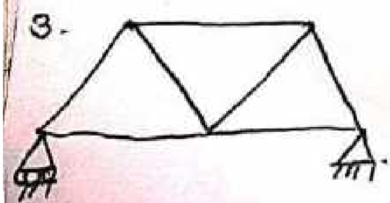


m = 5
r = 3
j = 4

$D_S = (m+r) - 2j$
$= (5+3) - 2(4)$
$= 8-8$
$= 0$

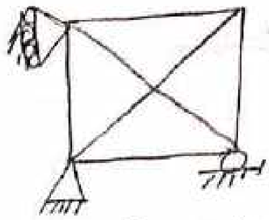
$D_k = 2j - r$
$= 2(4) - 3$
$= \underline{\underline{5}}$

3.



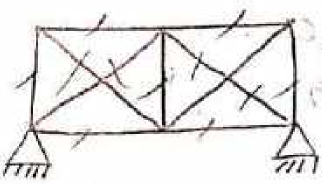
$D_S = (m+r) - 2j$
$= (7+3) - 2(5)$
$= 10-10$
$= 0$

$D_k = 2j - r$
$= 10 - 3$
$= 7$



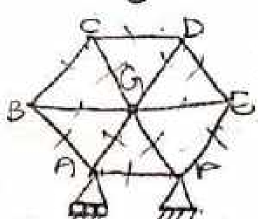
$D_S = (m+r) - 2j$
$= (6+4) - 2(4)$
$= 10-8$
$= 2$

$D_k = 2j - r$
$= 2(4) - 4$
$= 8-4$
$D_k = \underline{\underline{4}}$



$D_S = (m+r) - 2j$
$= (11+4) - 2(6)$
$= 15-12$
$= 3$

$D_k = 2j - r$
$= 2(6) - 4$
$= 12-4$
$= 8$



$D_S = (m+r) - 2j$
$= (12+3) - 2(7)$
$= 15 - 14 = 1$

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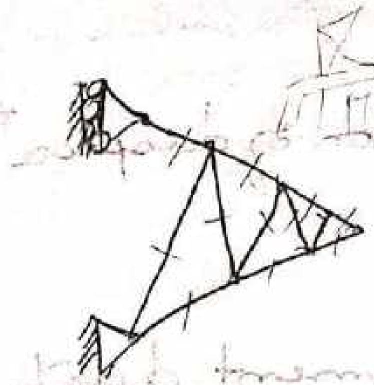
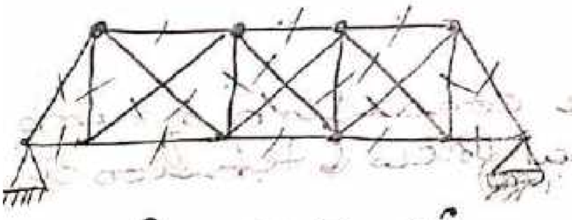
$$D_K = 2j - r$$
$$= 2(7) - 3$$
$$= 14 - 3$$
$$= 11$$



$$D_s = (m+r) - 2j$$
$$= (12+3) - 2(8)$$
$$= 15 - 16$$
$$= -1$$

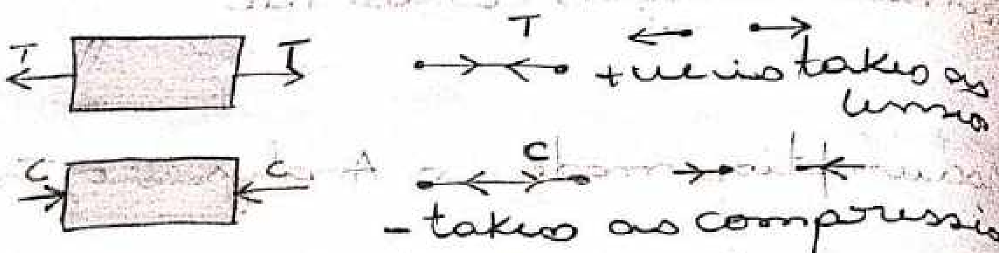
$$D_K = 2j - r$$
$$= 2(8) - 3$$
$$= 16 - 3$$
$$= \underline{13}$$



$$D_s = m + r - 2j$$
$$= 20 + 3 - 2(10)$$
$$= 23 - 20$$
$$= 3$$

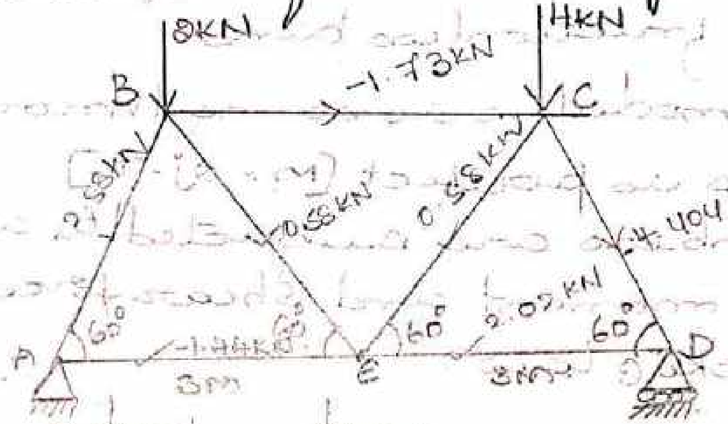
$$D_K = 2j - r$$
$$= 2(10) - 3$$
$$= 20 - 3$$
$$= 17$$

The forces are compressive in nature if they are pushing [acting towards the joint]

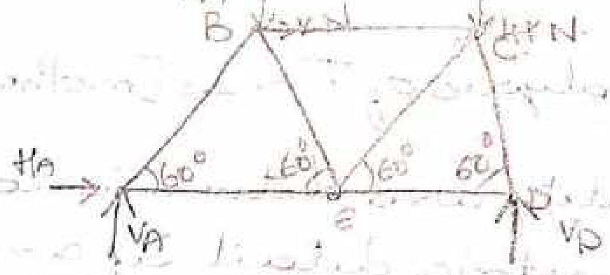


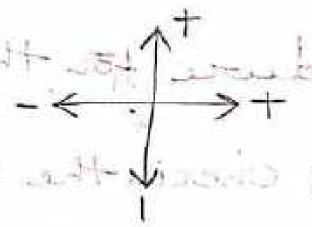
Problem

Calculate the member forces using method of joints



Reactions





$\Sigma V = 0$

$V_A + V_D - 2 - 4 = 0$

$V_A + V_D = 6$

$\Sigma H = 0$

$\boxed{H_A = 0}$

$\Sigma M_A = 0$

$(V_A \times 0) + (H_A \times 0) + (2 \times 1.5) + (4 \times 4.5) + (V_D \times 6) = 0$

$\boxed{V_D = 3.5\,\text{kN}}$

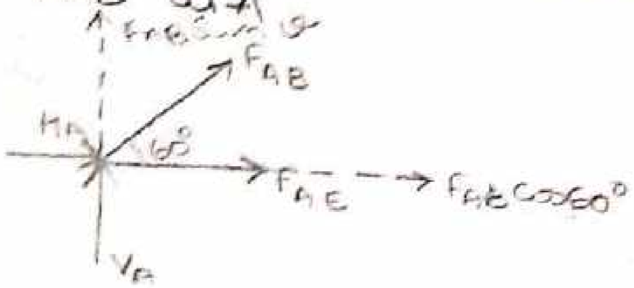
$V_A = 6 - V_D$

$V_A = 6 - 3.5$

$\boxed{V_A = 2.5\,\text{kN}}$

Scanned by CamScanner

Consider FBD at A



$\Sigma V = 0$

$+V_A + F_{AB}\sin 60^\circ = 0$

$2.5 + F_{AB}(0.866) = 0$

$\boxed{F_{AB} = -2.88\ kN\ (C)}$

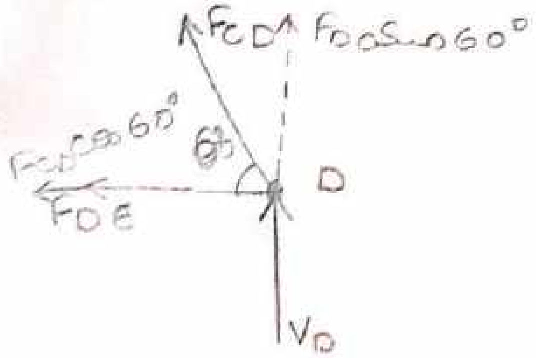
$\Sigma H = 0$

$H_A + F_{AB}\cos 60^\circ + F_{AE} = 0$

$0 (-2.88)\cos 60^\circ = -F_{AE}$

$\boxed{F_{AE} = 1.44 kN (T)}$

Consider FBD at D



$\Sigma V = 0$

$+V_D + F_{DC}\sin 60^\circ = 0$

$2.5 + F_{DC}(0.866) = 0$

$F_{DC}(0.866) = -2.5$

$\boxed{F_{DC} = -4.04 kN C}$

$\Sigma H = 0$

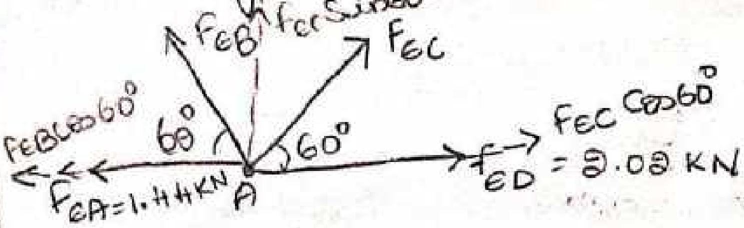
$F_{DE} + F_{DC}\cos 60^\circ = 0$

$F_{DE} = -F_{DC}\cos 60^\circ$

$F_{DE} = -[-4.04]\cos 60^\circ$

$\boxed{F_{DE} = 2.02 KN (T)}$

Consider joint at E



A to 491

$$\begin{array}{r} 2.02 \\ 1.44 \\ \hline \end{array}$$

$\Sigma V = 0$

$F_{EB}\sin 60^\circ + F_{EC}\sin 60^\circ = 0$

$F_{EB}\sin 60^\circ = -F_{EC}\sin 60^\circ$

$F_{EB} = -F_{EC}$

$F_{ee} = \frac{29}{50} = 0.58 KN$

$\boxed{F_{EB} = 0.58 KN}$

$\Sigma H = 0$

$-F_{EA} - F_{EB}\cos 60^\circ + F_{ED} + F_{EC}\cos 60^\circ = 0$

$-F_{EA} + F_{ED} = F_{EB}\cos 60^\circ + F_{EC}\cos 60^\circ$

$= -1.44 + 2.02 - F_{EB}\cos 60^\circ + F_{EC}\cos 60^\circ$

$0.58 - F_{EB}\cos 60^\circ + F_{EC}\cos 60^\circ$

$F_{EC}\cos 60^\circ - F_{EB}\cos 60^\circ = -0.58$

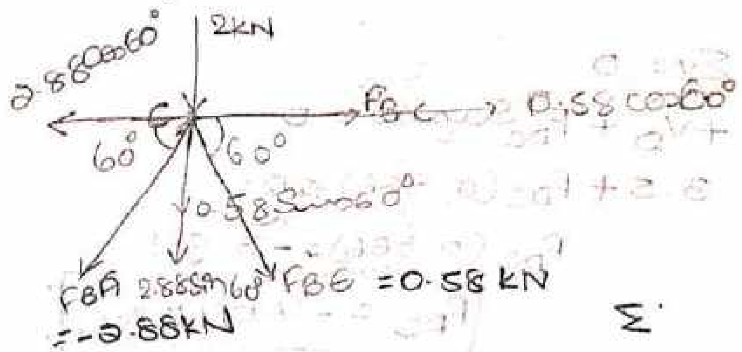
$0.58 - [-F_{EC}]\cos 60^\circ + F_{EC}\cos 60^\circ =$

$0.58 + 2F_{EC}\cos 60^\circ = 0$

$\boxed{F_{EC} = -0.58}$

$F_{EB} = 0.58 KN$

Consider joint B.



Σ

$\Sigma V = 0$

$-2 - 0.58\sin 60^\circ - 2.88\sin 60^\circ = 0$

$\Sigma H = 0$

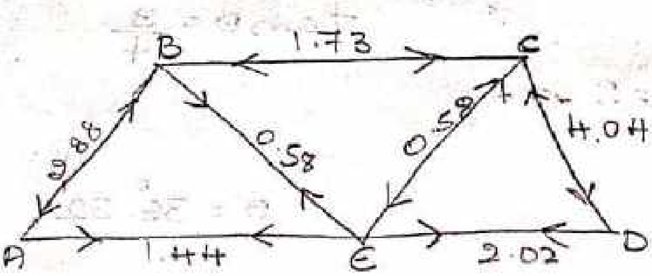
$F_{BC} + 0.58\cos 60^\circ + 2.88\cos$

$F_{BC} - 1.15 = 0$

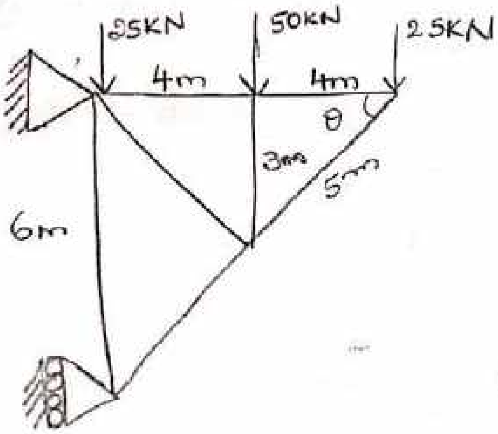
$F_{BC} = 1.73 KN$

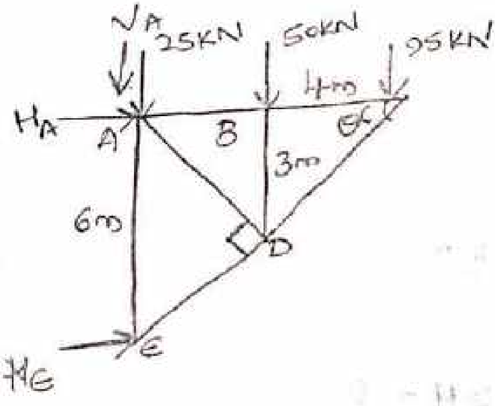
member	Force	Nature
AE	1.44	T
ED	2.02	T
BE	0.58	T
CD	4.04	C
AB	2.58	C
BC	1.73	C.
EB	-0.58	C

Calculated the member forces :



2.





$\tan\theta = \frac{adj}{hyp}$

$= \frac{4}{3}$

$\theta = \tan^{-1}(4/3)$

$\theta = 53°7'48.37''$

$\Sigma H = 0$

$+H_A + H_E = 0$

$H_A = -H_E$

$\Sigma V = 0$

$-V_A - 25 - 50 - 25 = 0$

$V_A = -100KN$

$\Sigma M_A = 0$

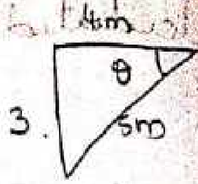
$(V_A \times 0)\ 50 \times 4 + 25 \times 8 - H_E \times 6 = 0$

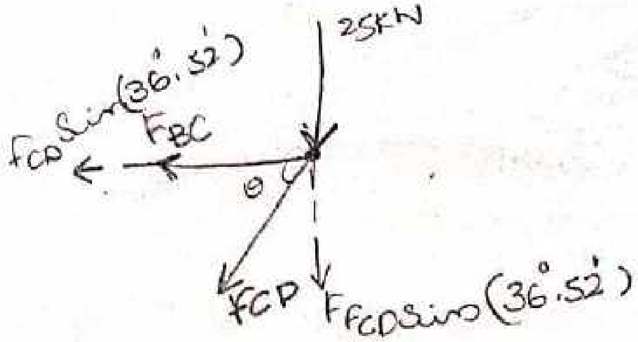
$\frac{400}{6} = H_E$

$\boxed{H_E = 66.66kN}$ $\quad H_A = -66.66kN$

Note; Since no external horizontal forces are given the values H_A and H_E is "0" $\therefore$ $H_A = H_E = 0$

Consider a joint C





$\tan\theta = \frac{3}{4}$

$\theta = \sin^{-1}[3/5]$.

$\theta = 36°.52'$

$\Sigma H = 0$

$-F_{CD}\sin(36°52') - F_{BC} = 0$

$F_{BC} = F_{CD}\sin(36°52')$

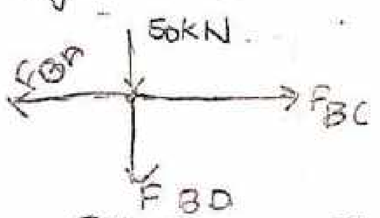
$\Sigma V = 0$

$-25 - F_{CD}\sin(36°.52') = 0$

$F_{CD} = -41.67(C)$

$F_{BC} = -33.340\,kN\ (C)$.

Consider joint - B



$\Sigma V = 0$

$-50kN - F_{BD} = 0$

$F_{BD} = -50\,kN$

$\Sigma H = 0$

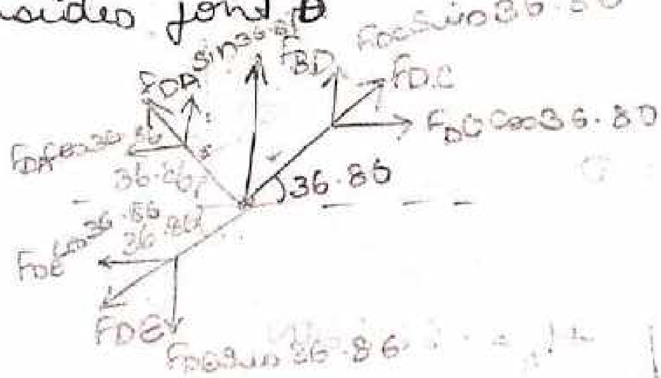
$F_{BC} - F_{BA} = 0$

$-F_{BA} = -F_{BC}$

$-F_{BA} = -33.34$

$F_{BA} = 33.34\,kN$

Consider joint D



$\Sigma H = F_{DC}\cos(36.86) - F_{DA}\sin(36.86) - F_{DE}\cos 36.86$

$= 41.67\cos(36.86') - F_{DA}\sin(36.86') - F_{DE}[\cos 36.86'] = 0$

$+33.338 = +F_{AD}\sin(36.86°) + F_{DE}\cos(36.86')$

$-F_{AD} - F_{DE} = +\dfrac{33.338}{\cos(36.86°)}$. $\quad F_{AD} + F_{DE} = -41.66kN$

$\Sigma V = 0$

$F_{BD} + F_{DA}\sin(36.86) + F_{DC}[\sin 36.86] - F_{DE}[\sin 36.86] = 0$

$-50 + F_{DA}\sin(36.86) + [-41.67]\sin(36.86) - F_{DE}(\sin 36.86) = 0$.

$F_{DA}\sin[36.86] - F_{DE}\sin[36.86] = 50 + 41.67\sin 36.86$

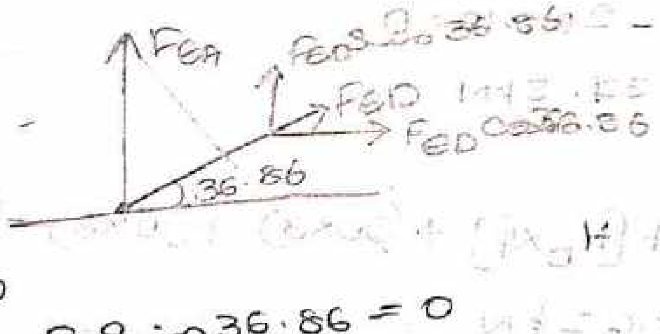
$F_{DA}\sin[36.86] - F_{DC}(\sin 36.86] = 74.996$.

$F_{DA} - F_{DE} = \dfrac{74.996}{\sin(36.86)}$

$F_{DA} - F_{DE} = 125.027$

$\boxed{F_{DA} = 41.68\ kN\ T}$ $\quad$ $\boxed{F_{DE} = -83.341\ kN\ C}$

Consider the joint A

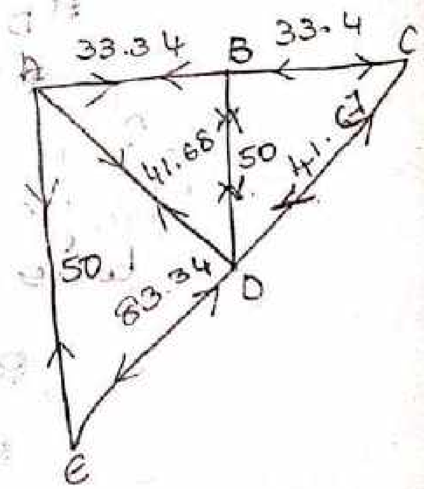


$\Sigma V = 0$

$+F_{EA} + F_{ED}\sin 36.86 = 0$

$F_{EA} = -83.341\sin 36.86$

$\boxed{F_{EA} = 49.99\ kN / 50\ kN\ (T)}$



For member	force	Nature
CD	41.67	(C)
BC	33.34	(C)
BD	50	(C)
BA	33.34	
DA	41.68	(T)
DE	83.341	(C)
EA	49.99	(T).

MODULE-02

INFLUENCE LINE DIAGRAMS

ROLLING LOAD AND INFLUENCE LINES

1 Introduction: Variable Loadings

So far in this course we have been dealing with structural systems subjected to a specific set of loads. However, it is not necessary that a structure is subjected to a single set of loads all of the time. For example, the single-lane bridge deck in Figure1 may be subjected to one set of a loading at one point of time (Figure1a) and the same structure may be subjected to another set of loading at a different point of time. It depends on the number of vehicles, position of vehicles and weight of vehicles. The variation of load in a structure results in variation in the response of the structure. For example, the internal forces change causing a variation in stresses that are generated in the structure. This becomes a critical consideration from design perspective, because a structure is designed primarily on the basis of the intensity and location of maximum stresses in the structure. Similarly, the location and magnitude of maximum deflection (which are also critical parameters for design) also become variables in case of variable loading. Thus, multiple sets of loading require multiple sets of analysis in order to obtain the critical response parameters.

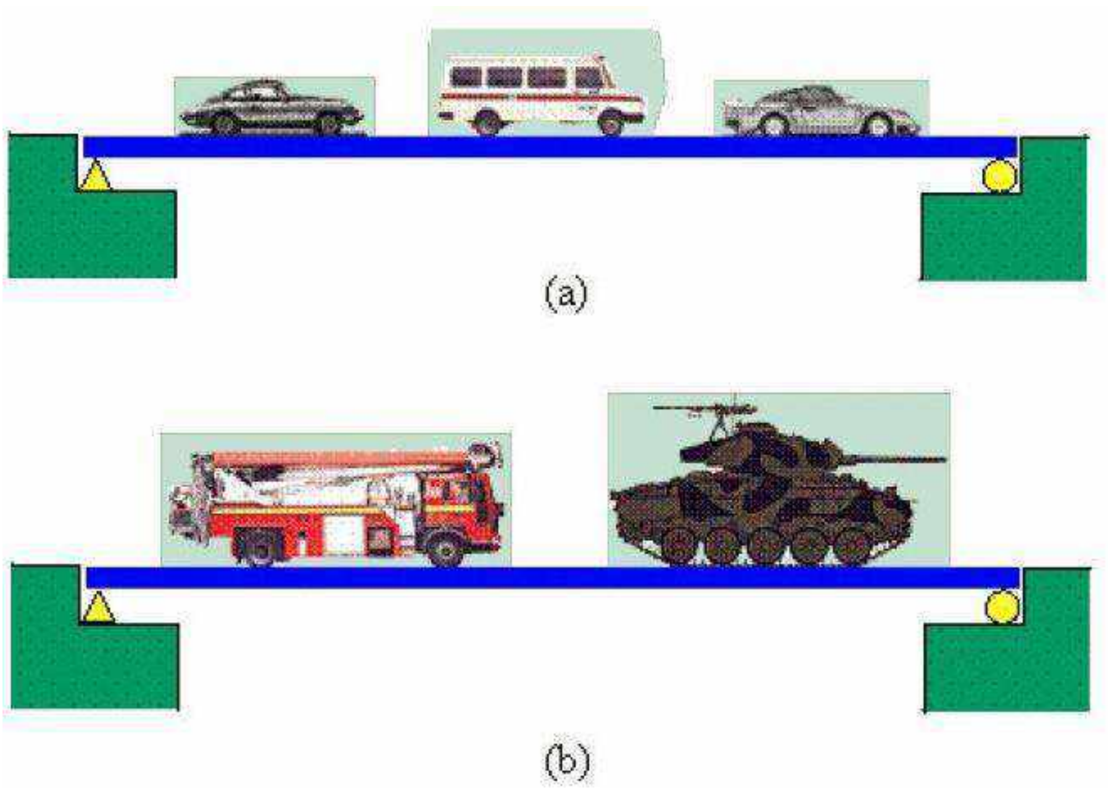


Figure 1 Loading condition on a bridge deck at different points of time

Influence lines offer a quick and easy way of performing multiple analyses for a single structure. Response parameters such as *shear force* or *bending moment at a point* or *reaction at a support* for several load sets can be easily computed using influence lines.

For example, we can construct influence lines for (shear force at B) or (bending moment at) or (vertical reaction at support D) and each one will help us calculate the corresponding response parameter for different sets of loading on the beam *AD* (Figure 2).

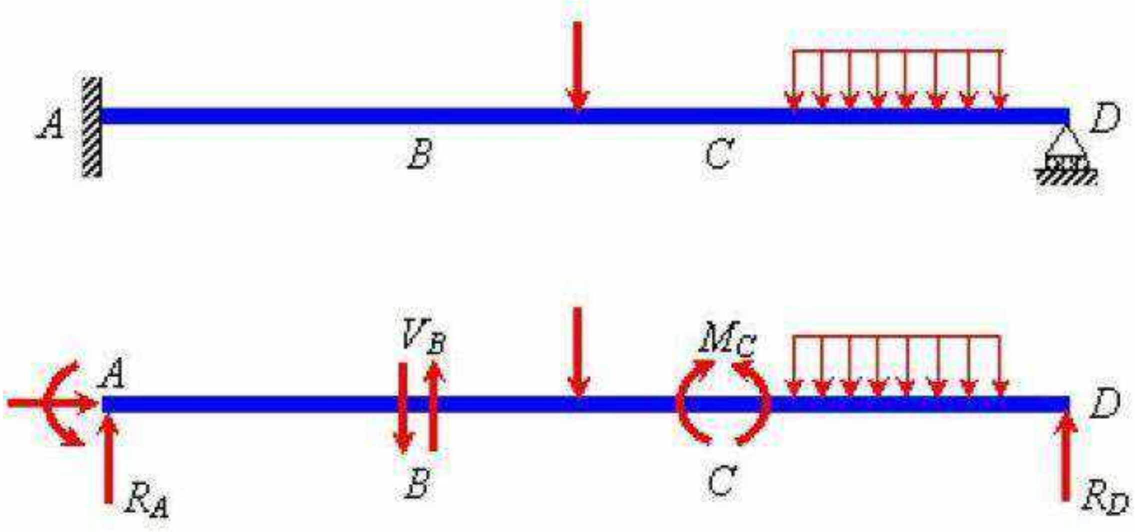
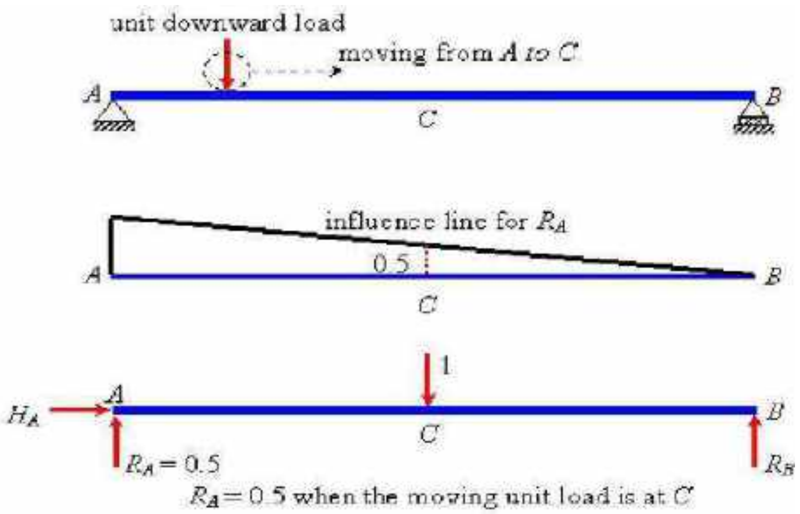


Figure 2 Different response parameters for beam *AD*

An influence line is a diagram which presents the variation of a certain response parameter due to the variation of the position of a unit concentrated load along the length of the structural member. Let us consider that a unit downward concentrated force is moving from point *A* to point *B* of the beam shown in Figure 3a. We can assume it to be a wheel of unit weight moving along the length of the beam. The magnitude of the vertical support reaction at *A* will change depending on the location of this unit downward force. The influence line for (Figure3b) gives us the value of for different locations of the moving unit load. From the ordinate of the influence line at *C*, we can say that when the unit load is at point *C* .



Thus, an influence line can be defined as a curve, the ordinate to which at any abscissa gives the value of a particular response function due to a unit downward load acting at the point in the structure corresponding to the abscissa. The next section discusses how to construct influence lines using methods of equilibrium.

2 Construction of Influence Lines using Equilibrium Methods

The most basic method of obtaining influence line for a specific response parameter is to solve the static equilibrium equations for various locations of the unit load. The general procedure for constructing an influence line is described below.

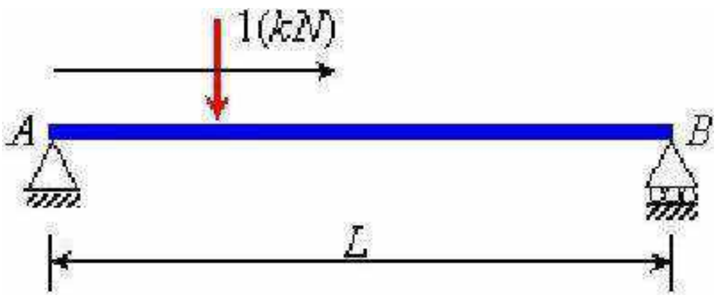
1. Define the positive direction of the response parameter under consideration through a free body diagram of the whole system.

2..For a particular location of the unit load, solve for the equilibrium of the whole system and if required, as in the case of an internal force, also for a part of the member to obtain the response parameter for that location of the unit load.This gives the ordinate of the influence line at that particular location of the load.

3. Repeat this process for as many locations of the unit load as required to determine the shape of the influence line for the whole length of the member. It is often helpful if we can consider a generic location (or several locations) *x* of the unit load.

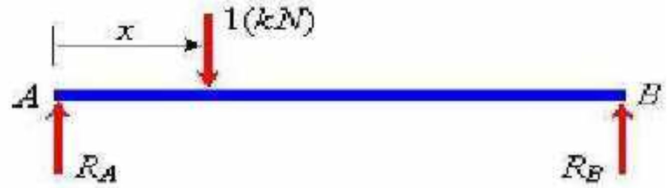
4. Joining ordinates for different locations of the unit load throughout the length of the member, we get the influence line for that particular response parameter. The following three examples show how to construct influence lines for a support reaction, a shear force and a bending moment for the simply supported beam *AB* .

Example 1 Draw the influence line for (vertical reaction at *A*) of beam *AB* in Fig.1



Solution:

Free body diagram of *AB* :

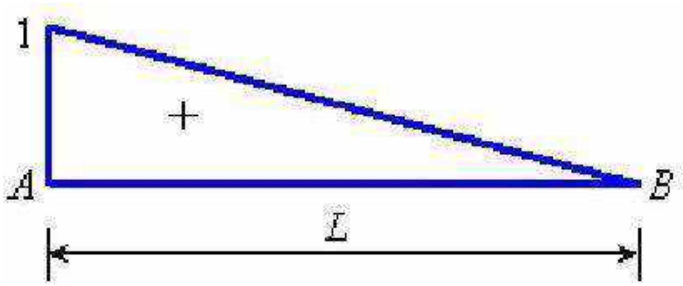


$$\sum F_y = 0 \Rightarrow R_A = 1 - R_B$$

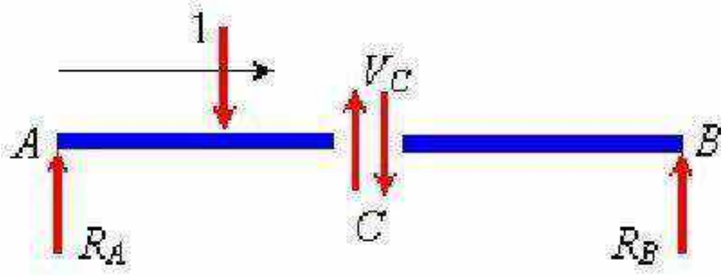
$$\sum M\,(about\,B) = 0 \Rightarrow R_A(L) = 1(L - x)$$

$$\Rightarrow R_A = 1 - \frac{x}{L}$$

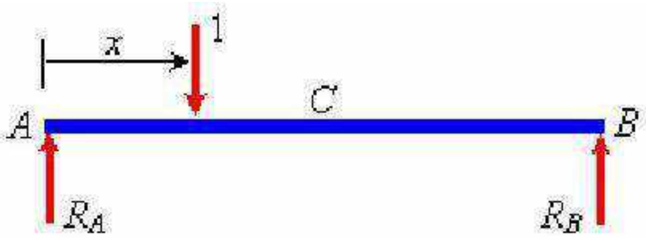
So the influence line of :



Example 2 Draw the influence line for (shear force at mid point) of beam *AB* in Fig.2.

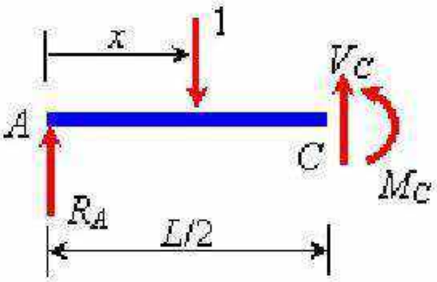


Solution:



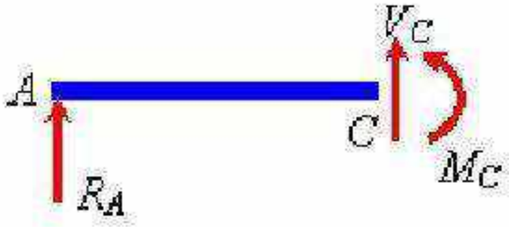
$$\sum M(about\ B) = 0 \Rightarrow R_A = 1 - \frac{x}{L}$$

For $x < \frac{L}{2}$



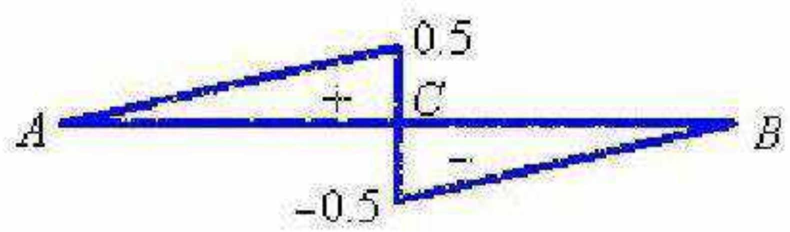
$$\sum F_y = 0 \Rightarrow V_C = 1 - R_A = \frac{x}{L}$$

For $x > \frac{L}{2}$

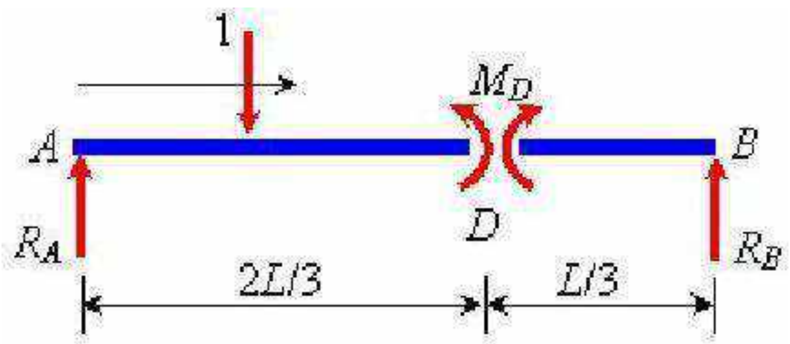


$$\sum F_y = 0 \Rightarrow V_C = -R_A = \frac{x}{L} - 1$$

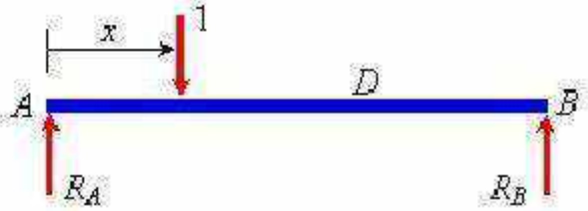
So the influence line for V_C:



Example 3 Draw the influence line for (bending moment at) for beam *AB* in Fig.3.

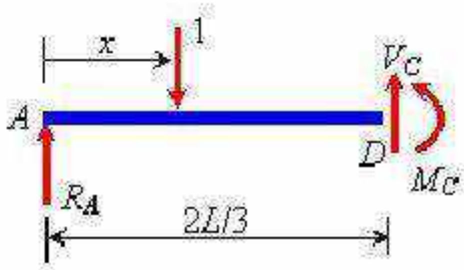


Solution:



$$\sum M(about\ B) = 0 \Rightarrow R_A = 1 - \frac{x}{L}$$

For $x < \frac{2L}{3}$

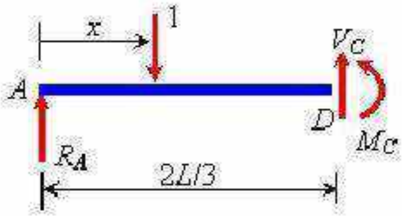


$$\sum M(about\ D) = 0$$

$$\Rightarrow M_D = R_A\left(\frac{2L}{3}\right) - 1\left(\frac{2L}{3} - x\right) = \left(1 - \frac{x}{L}\right)\left(\frac{2L}{3}\right) - \frac{2L}{3} + x$$

$$= -\frac{2x}{3} + x = \frac{x}{3}$$

For $x < \frac{2L}{3}$

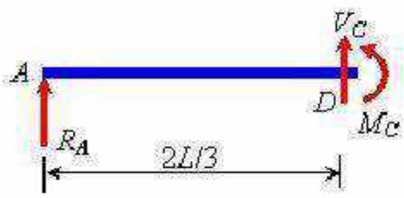


$$\sum M(about\ D) = 0$$

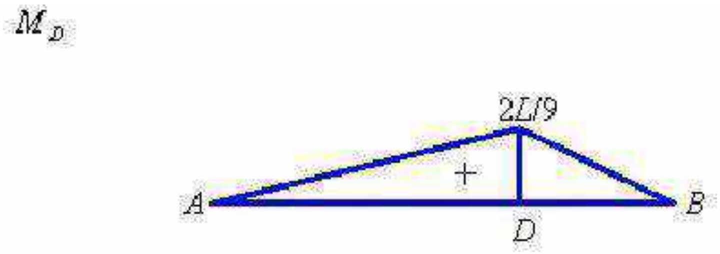
$$\Rightarrow M_D = R_A\left(\frac{2L}{3}\right) - 1\left(\frac{2L}{3} - x\right) = \left(1 - \frac{x}{L}\right)\left(\frac{2L}{3}\right) - \frac{2L}{3} + x$$

$$= -\frac{2x}{3} + x = \frac{x}{3}$$

For $x > \frac{2L}{3}$



$$\sum M(about\ D) = 0 \Rightarrow M_D = R_A\left(\frac{2L}{3}\right) = \frac{2L}{3} - \frac{2x}{3}$$



Similarly, influence lines can be constructed for any other support reaction or internal force in the beam. However, one should note that equilibrium equations will not be sufficient to obtain influence lines in indeterminate structures, because we cannot solve for the internal forces/support reactions using only equilibrium conditions for such structures.

3 Use of Influence Lines

In this section, we will illustrate the use of influence lines through the influence lines that we have obtained in Section 2. Let us consider a general case of loading on the simply supported beam (Figure 4a) and use the influence lines to find out the response parameters for their loading. We can consider this loading as the sum of three different loading conditions, (A), (B) and (C) (Figure 4b), each containing only one externally applied force.

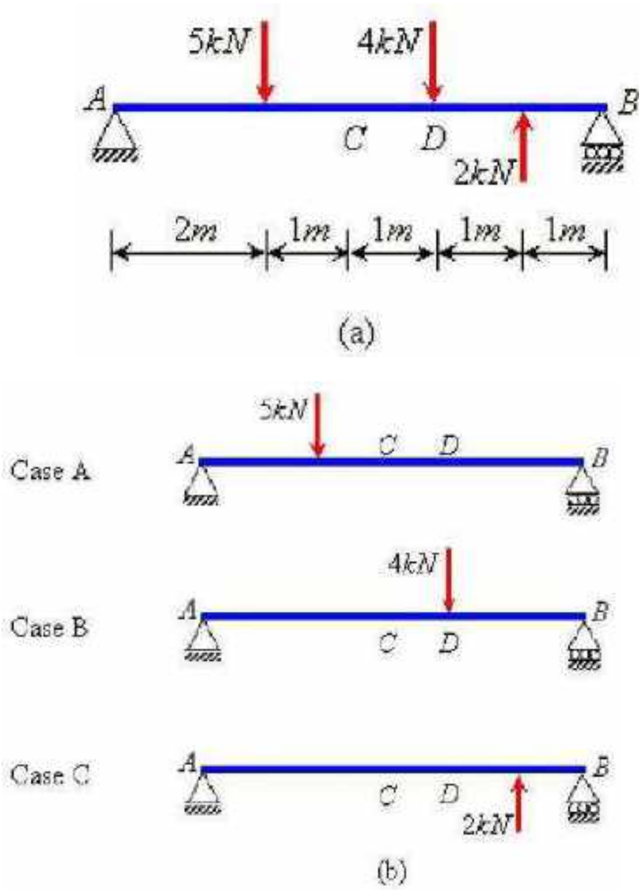


Figure4: Application of influence lines for a general loading: (a) all the loads, and (b) the general loading is divided into single force systems

For loading case (A), we can find out the response parameters using the three influence lines. Ordinate of an influence line gives the response for a unit load acting at a certain point.

Therefore, we can multiply this ordinate by the magnitude of the force to get the response due to the real force at that point. Thus

$$R_A = 5kN \times (\text{ordinate of the } IL \text{ of } R_A \text{ at } x = 2m) = 5(1 - 2/6) = 3.33kN$$

$$V_C = 5kN \times (\text{ordinate of the } IL \text{ of } V_C \text{ at } x = 2m) = 5(2/6) = 1.67kN$$

$$M_D = 5kN \times (\text{ordinate of the } IL \text{ of } M_D \text{ at } x = 2m) = 5(2/3) = 3.33kNm$$

Similarly, for loading case (B):

$$R_A = 4kN \times (\text{ordinate of the } IL \text{ of } R_A \text{ at } x = 4m) = 4(1 - 4/6) = 1.33kN$$

$$V_C = 4kN \times (\text{ordinate of the } IL \text{ of } V_C \text{ at } x = 4m) = 4(4/6 - 1) = -1.33kN$$

$$M_D = 4kN \times (\text{ordinate of the } IL \text{ of } M_D \text{ at } x = 4m) = 4(2 \times 6/9) = 5.33kNm$$

And for case (C),

$$R_A = -2kN \times (\text{ordinate of the } IL \text{ of } R_A \text{ at } x = 5m) = -2(1 - 5/6) = -0.33kN$$

$$V_C = -2kN \times (\text{ordinate of the } IL \text{ of } V_C \text{ at } x = 5m) = -2(5/6 - 1) = 0.33kN$$

$$M_D = -2kN \times (\text{ordinate of the } IL \text{ of } M_D \text{ at } x = 5m) = -2(2 \times 6/3 - 2 \times 5/3) = -1.33kNm$$

By the theory of superposition, we can add forces for each individual case to find the response parameters for the original loading case (Figure4a). Thus, the response parameters in the beam *AB* are:

$$R_A = (3.33 + 1.33 - 0.33)kN = 4.33kN$$

$$V_C = (1.67 - 1.33 + 0.33)kN = 0.67kN$$

$$M_D = (3.33 + 5.33 - 1.33)kN = 7.33kNm$$

One should remember that the method of superposition is valid only for linear elastic cases with small displacements only. So, prior to using influence lines in this way it is necessary to check that these conditions are satisfied.

It may seem that we can solve for these forces under the specified load case using equilibrium equations directly, and influence lines are not necessary. However, there may be requirement for obtaining these responses for multiple and more complex loading cases. For example, if we need to analyse for ten loading cases, it will be quicker to find only three influence lines and not solve for ten equilibrium cases.

The most important use of influence line is finding out the location of a load for which certain response will have a maximum value. For example, we may need to find the location of a moving load (say a gantry) on a beam (say a gantry girder) for which we get the maximum bending moment at a certain point. We can consider bending moment at point *D* of

Example3, where the beam *AB* becomes our gantry girder. Looking at the influence line of one can say that will reach its maximum value when the load is at point *D* . Influence lines can be used not only for concentrated forces, but for distributed forces as well, which is discussed in the next section.

4 Using Influence Lines for Uniformly Distributed Load

Consider the simply-supported beam *AB* in Figure 6.5, of which the portion *CD* is acted upon by a uniformly distributed load of intensity *w/unit length* . We want to find the value of a certain response function *R* under this loading and let us assume that we have already constructed the influence line of this response function. Let the ordinate of the influence line at a distance *x* from support *A* be . If we consider an elemental length *dx* of the beam at a distance *x* from *A* , the total force acting on this elemental length is *wdx* . Since *dx* is infinitesimal, we can consider this force to be a concentrated force acting at a distance *x* .The contribution of this concentrated force *wdx* to *R* is:

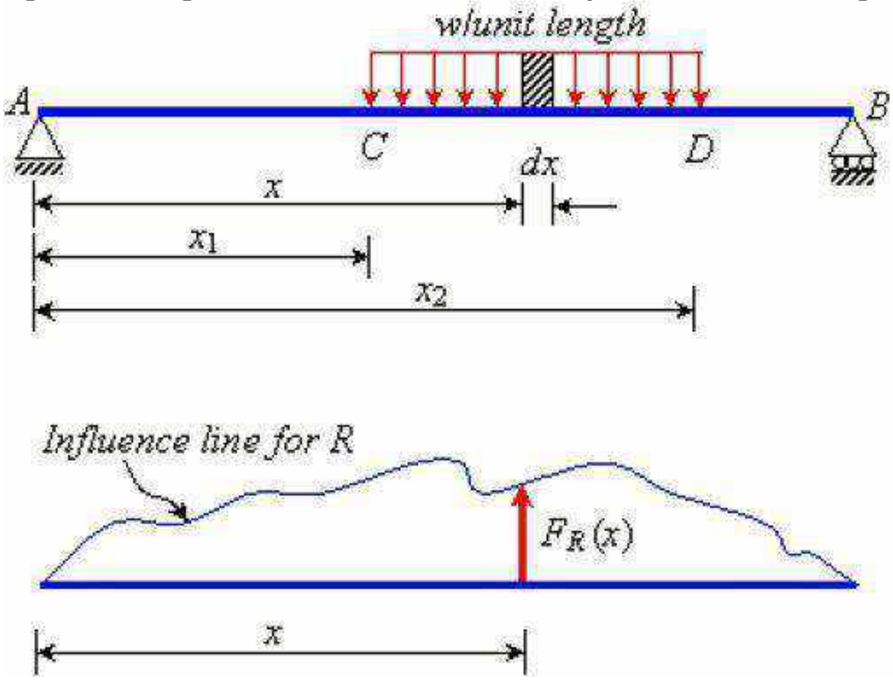
$$dR = (wdx)F_R(x)$$

Therefore, the total effect of the distributed force from point *C* to *D* is:

$$R = \int_C^D dR = \int_{x_1}^{x_2} wF_R(x)dx$$

$$= w\int_{x_1}^{x_2} F_R(x)dx = w \text{ (area under the influence line from } C \text{ to } D \text{)}$$

Figure 5 Using influence line for a uniformly distributed loading



Thus, we can obtain the response parameter by multiplying the intensity of the uniformly distributed load with the area under the influence line for the distance for which the load is acting. To illustrate, let us consider the uniformly distributed load on a simply supported beam (Figure 6). To find the vertical reaction at the left support, we can use the influence line for that we have obtained in Example 1. So we can calculate the reaction as:

$$R_A = 2kN/m \times \{0.5(3/4 + 1/4) \times 4m\} = 4kN$$

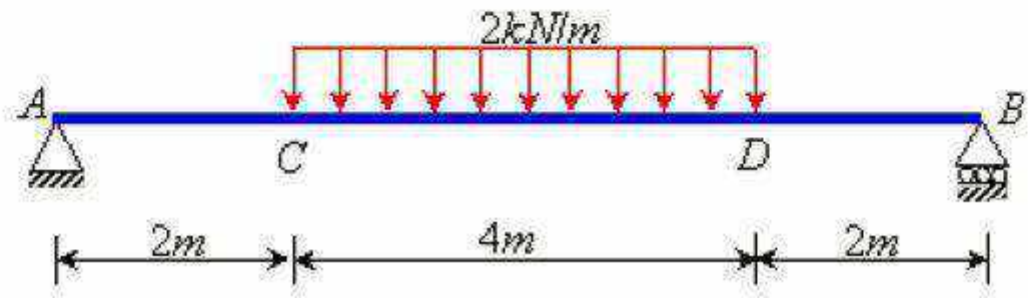


Figure 6.6 Uniformly distributed load acting on a beam

Similarly, we can find any other response function for a uniformly distributed loading using their influence lines as well. For non-uniformly distributed loading, the intensity *w* is not constant through the length of the distributed load. We can still use the integration formulation:

$$R = \int_C^D dR = \int_{x1}^{x2} wF_R(x)dx$$

However, we cannot take the intensity *w* outside the integral, as it is a function of *x* .

Influence Lines

"An Influence line is a graph showing for any beam, frame or truss, the variation of any force or displacement quantity (such as shear force, B.M, tension, deflection) for all positions of a moving unit load as it crosses the structure from end to the other"

For statically determinate structures the ILD [Influence Line Diagram] for force quantities are madeup of displaced configuration of the undeformed parts of the structures.

Eg:

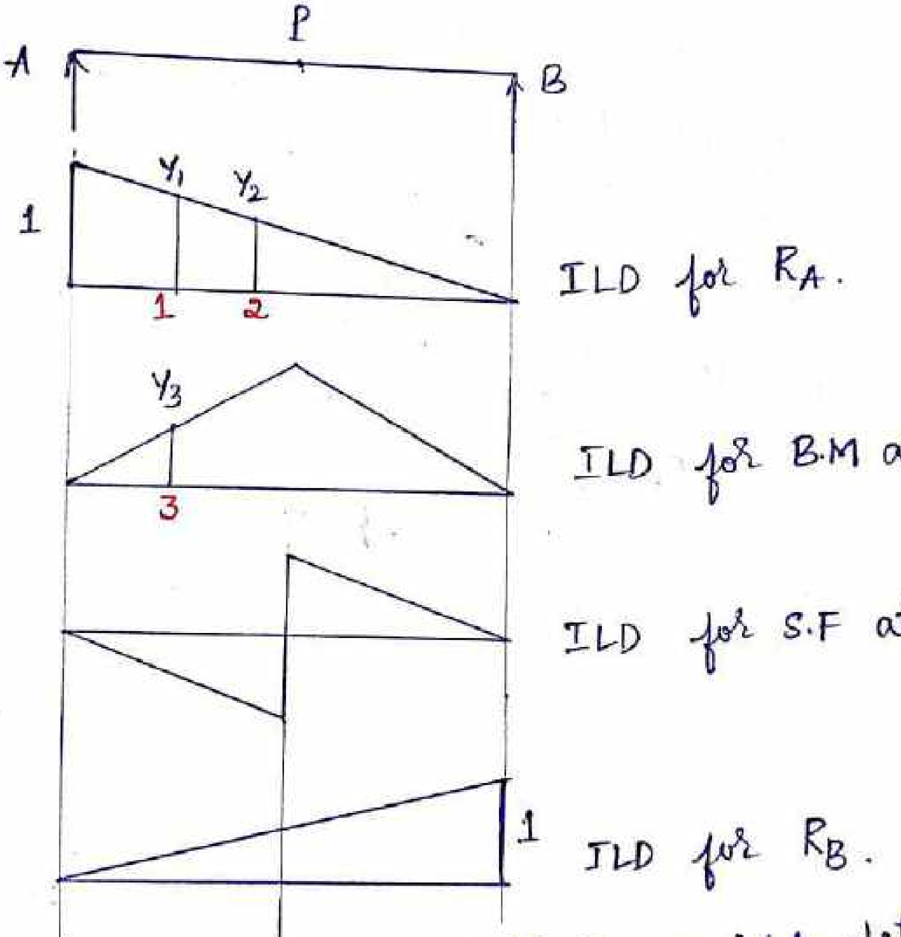


Fig : 01

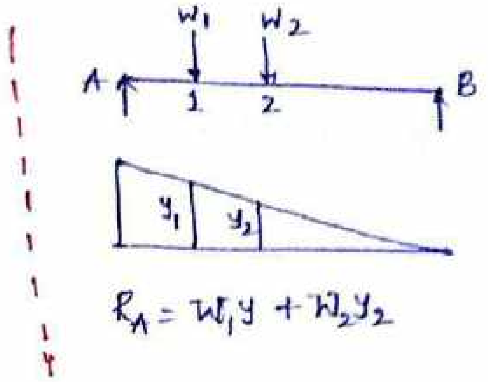
Fig : 02

Fig : 03

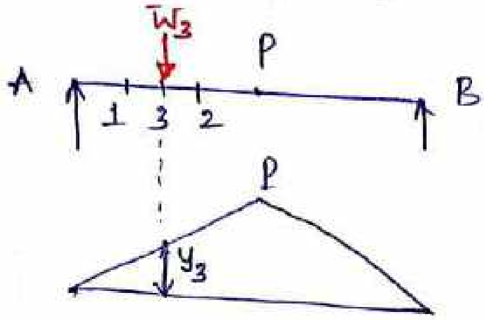
Fig : 04

Influence lines are very usefull in quickly determining the force component at any given point due to set of moving loads. Thus in fig: 01 Y_1 is the reaction at 'A' due to unit load at '1', Y_2 is the reaction at A due to unit load at '2'

And $[W_1 y_1 + W_2 y_2]$ is the reaction at 'A' due to loads W_1 at 1 & W_2 at 2. ②

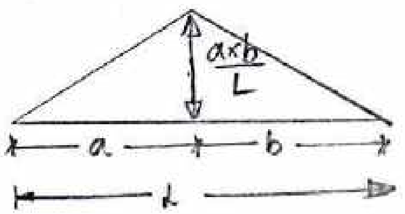


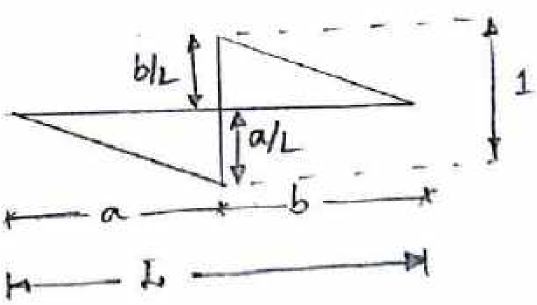
Extending the reasoning y_3 is the bending moment at P due to unit load at 3. $W_3 y_3$ is the BM at P due to a load W_3 at 3.

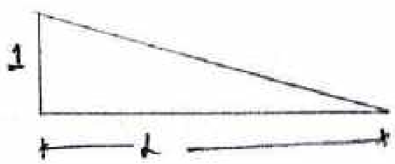


BM at P = $W_3 y_3$

Influence lines duly plotted can help us to get the total influence of all the loads on a span in inducing shear or bending moment at a point or a support B.M.
They can also help us to decide upon the strategic load positions to create maximum shear or bending moment at a given section for which the ILD are drawn.







03. ~~Consider~~ A simply supported beam of span 6m is traversed by a UDL of 8m long with intensity 15 kN/m draw the influence line diagram for.

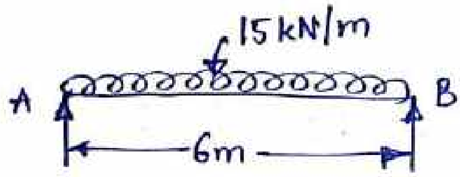
(i) Reaction at left support

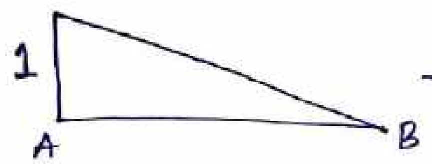
(ii) S.F at 2m from left support

(iii) B.M at 2m from left support

Find the maximum values of above quantities.

Soln: (i) Load position for Maximum reaction at left support is

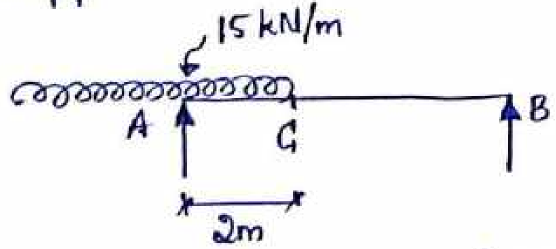




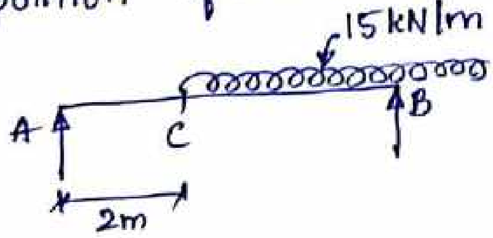
→ ILD for R_A

$\therefore R_A = 15 \times (\frac{1}{2} \times 6 \times 1) = \underline{\underline{45 \text{ kN}}}$

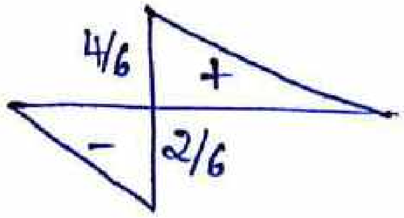
(ii) a) Load position for maximum -ve SF at 2m from left support.



b) Load position for maximum +ve SF at 2m i.e at C



ILD for SF at C (i.e at 2m from left support)



$\therefore$ Maximum -ve SF @ C $= 15\times(1/2\times 2\times 2/6)$

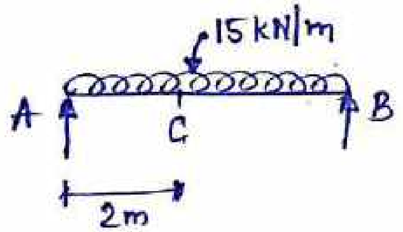
$= 5$ kN

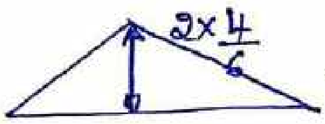
$\therefore$ Maximum +ve SF @ C $= 15\times(1/2\times 2\times 4/6)$

$= 10$ kN

(iii)

Load position for maximum BM at C.





→ ILD for M_C

$\therefore$ Maximum BM $= 15\times(1/2\times 6\times \frac{8}{6})$

$= 60$ kN-m

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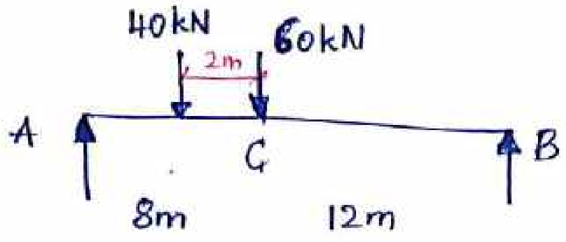
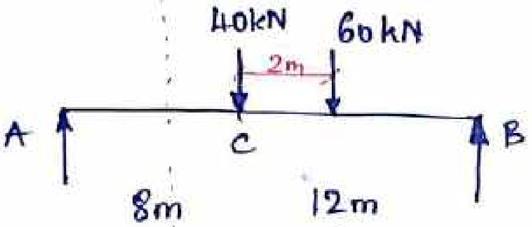
04. A beam has a span of 20m. draw influence line for BM and SF at a section 8m from the left support & determine the maximum BM & SF for this section due to two point loads 60kN & 40kN at a fixed distance of 2m apart rolling from left to right with 60kN load leading.

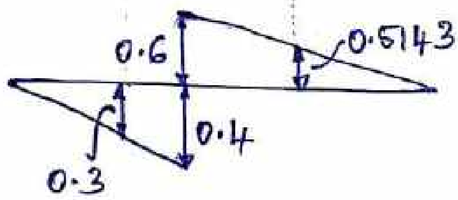
Soln.



Load position for maximum -ve SF.



Load position for maximum +ve SF



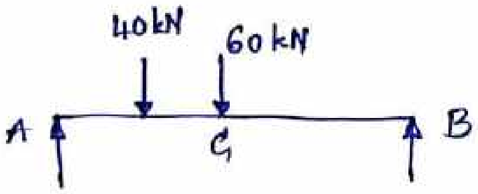
ILD for SF at C

Maximum -ve SF at C $= (40 \times 0.3) + (60 \times 0.4)$

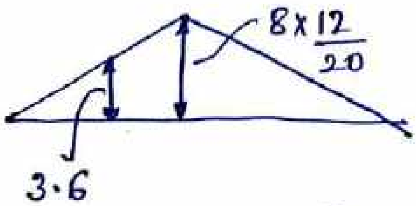
$= \underline{36 \text{ kN}}$

Maximum +ve SF at C $= (40 \times 0.6) + (60 \times 0.5134)$

$= \underline{54.80 \text{ kN}}$



→ Load position for max BM at C



ILD for BM @ C

B.M at C $= (60 \times 4.8) + (40 \times 3.6)$

$= \underline{432 \text{ kN-m}}$

01. A single rolling load of 100 kN moves on a girder of span 20m

a) Construct the influence line for

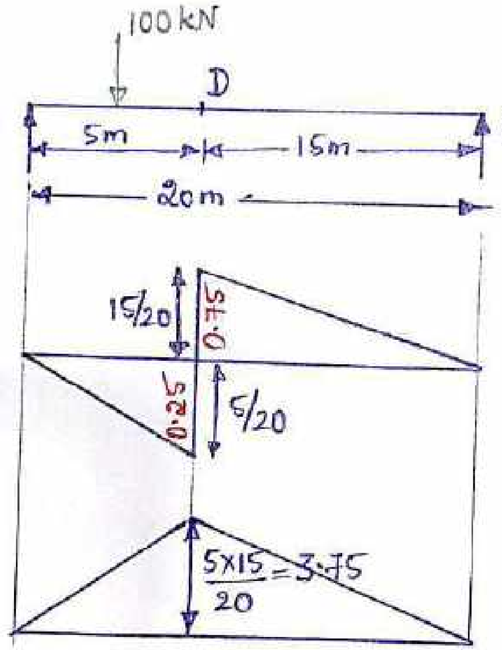
(i) Shear force &

(ii) B.M for a section 5m from the left support

b) Construct the influence line for points at which the maximum shear & maximum B.M develops. Determine these maximum values. (a & b)

Soln

a)



ILD for S.F @ D

(i) Maximum +ve S.F: By inspection of the ILD it is evident that maximum +ve S.F occurs when the load is placed just to the right of D.

∴ Max. +ve S.F = load × Ordinate = 100 × 0.75

= 75 kN

Maximum -ve SF: Maximum -ve SF occurs when the load is placed just to the left of D.

∴ Max. -ve SF = load × ordinate

= 100 × 0.25

= 25 kN

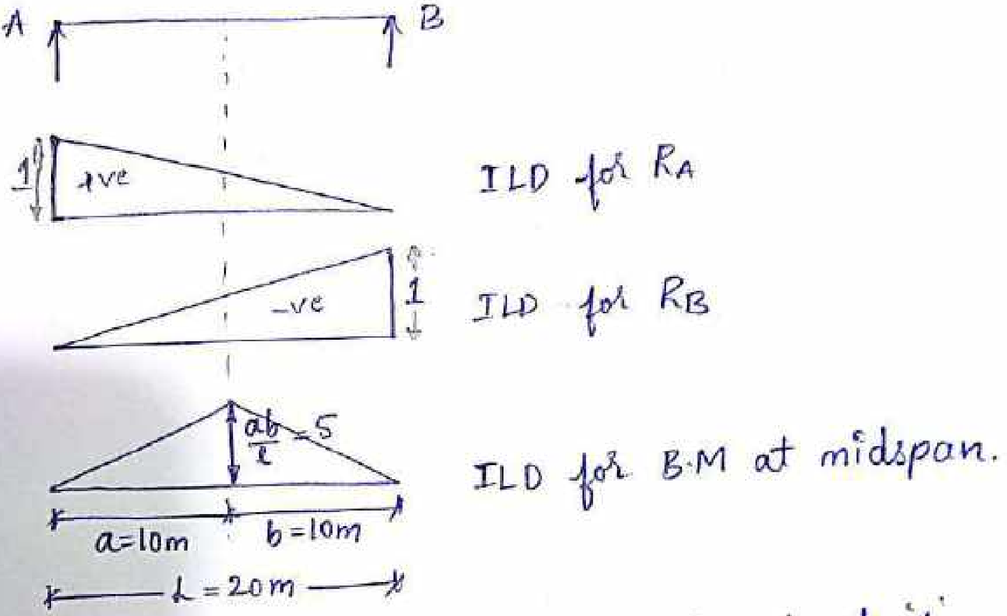
a) (ii) Maximum B.M : Maximum B.M occurs when the load is placed on the section 'D' itself.

$\therefore$ Max. B.M = load × Ordinate = 100 × 3.75

= 375 kN-m

b) Maximum +ve SF will occur at 'A' and maximum −ve S.F will occur at 'B'

Maximum B.M will occur at midspan. The ILD are shown below.



+ve S.F : Max. +ve SF occurs when load is placed at 'A'

$\therefore$ Max +ve S.F = load × Ordinate = 100 × 1 = 100 kN

−ve S.F : Max −ve SF occurs when load is placed at 'B'

$\therefore$ Max −ve S.F = load × Ordinate = 100 × −1 = −100 kN

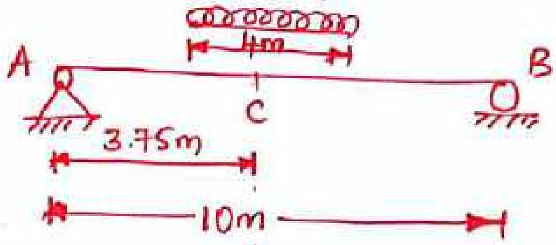
Maximum B.M : Maximum B.M occurs when the load is at midspan

Max. B.M = load × Ordinate

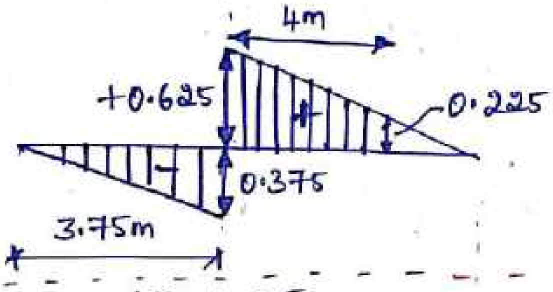
= 100 × 5

= 500 kN-m

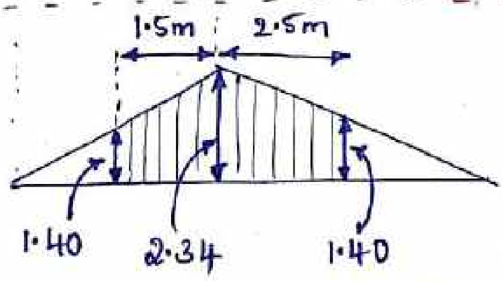
01. Determine the maximum shear force and moment at section C for the beam shown in fig. The beam is traversed by a uniformly distributed load of intensity 20 kN/m extended over a length of 4m. Indicate the sections that experience the absolute maximum shear and maximum moment.



Soln:



ILD for SF @ C



ILD for Moment @ C

From ILD for SF, it is obvious that the load should cover the left of section C for maximum negative shear.

$$\therefore \quad V_{max} = \frac{0.375}{2} \times 3.75 \times 20 = \underline{14.06\ kN\ (-ve)}$$

III^{ly}, the maximum positive shear force will occur when the load is placed to the right of section C, i.e

$$V_{max} = \tfrac{1}{2} \times (0.625 + 0.225) \times 4 \times 20 + \quad = \underline{68.0\ kN\ (+ve)}$$

$$M_{C_{max}} = \tfrac{1}{2}(1.4 + 2.34)(1.5)(20) + \tfrac{1}{2}(1.4 + 2.34)(2.5)(20)$$

$$= \underline{149.6\ kN\text{-}m}$$

From a knowledge of the influence lines for SF & Moment, it can be said that the absolute maximum SF will occur next to the support points and the obsoulute maximum moment occurs at centre of span. The values given may be verified.

$$V_{max} = \underline{\underline{\pm 64.0\ kN}} \quad \& \quad M_{max} = \underline{\underline{160.\ kN\text{-}m}}$$

Q2 Define Influence line diagram & mention its applica

"ILD is the graphical representation of variation of reaction, shear force, Bending moment when unit loa moves over the beam from left to right or vice versa"

<u>Uses</u>

(i) It is used to determine reaction results, maximum shear force & maximum bending moment on a beam when unit load moves on beam.

(ii) ILD are used to determine the maximum results & position of the moving loads

(iii) To determine absolute maximum shear & absolute maximum B.M.

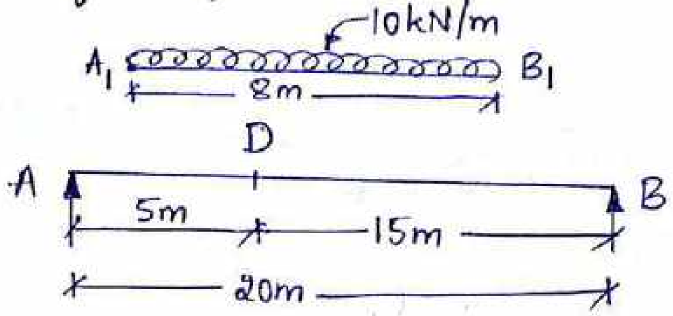
(iv) The ILD can be used for all types of loads

(v) Suitable for stationary & moving load

(vi) Shear stress reversal in panels can be assesed & in trusses

([illegible]

Q3 Draw the influence line diagram for S.F & B.M for a section at 5m from the left hand support of a simply supported beam of span 20m. Hence calculate the maximum B.M & S.F at the section due to uniformly distributed rolling load of length 8m and intensity 10kN/m.

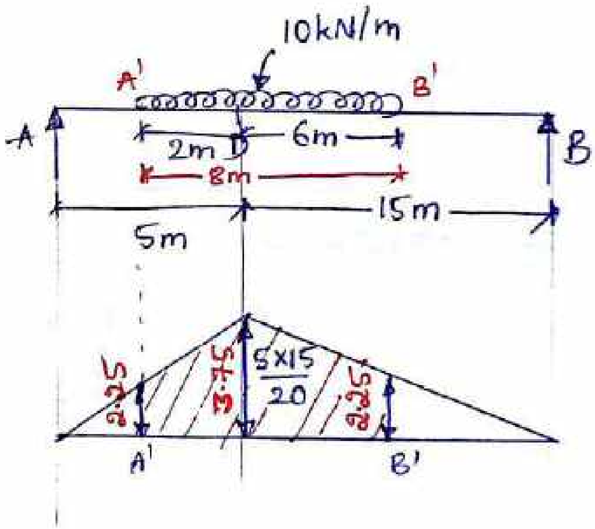


Soln: a) Maximum B.M: Max. B.M at D due to UDL shorter than the span occurs when the section divides the load in the same ratio as it divides the span.

i.e $\dfrac{A_1D}{B_1B_1} = \dfrac{AD}{AB} = \dfrac{5}{20} = 0.25$

$\therefore\ A_1D_1 = 8 \times 0.25 = 2m.$

$B_1D = 6m$



Ordinate under $A' = \dfrac{3.75}{5} \times 3 = 2.25$

—— " —— $B' = \dfrac{3.75}{15} \times 9 = 2.25$

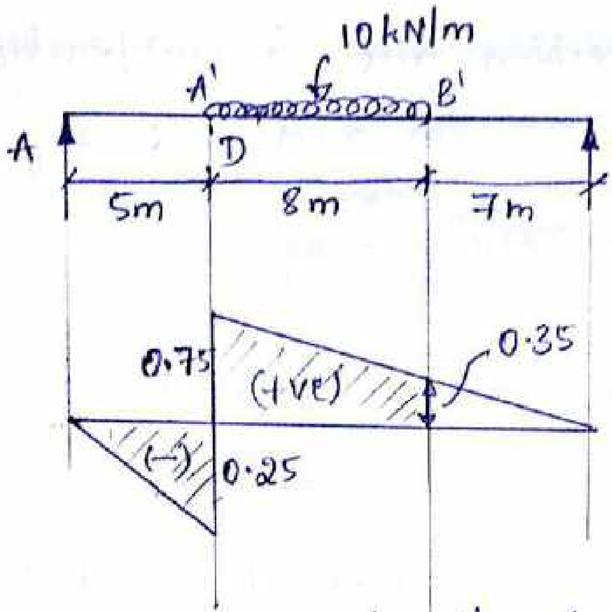
$\therefore$ Maximum B.M = intensity of load × Area of ILD under load

$= 10 \times \left(\dfrac{3.75 + 2.25}{2}\right) \times 8 = \underline{240\ kN\text{-}m}$

(or)

$= 10 \times \left[\left(\frac{1}{2} \times 8 \times (3.75 - 2.25)\right) + (8 \times 2.25)\right] = \underline{240\ kN\text{-}m}$

b) Maximum +ve S.F : Maximum +ve SF occurs when the tail of the UDL is at D. as it traverses from left to right.



Ordinate under B'

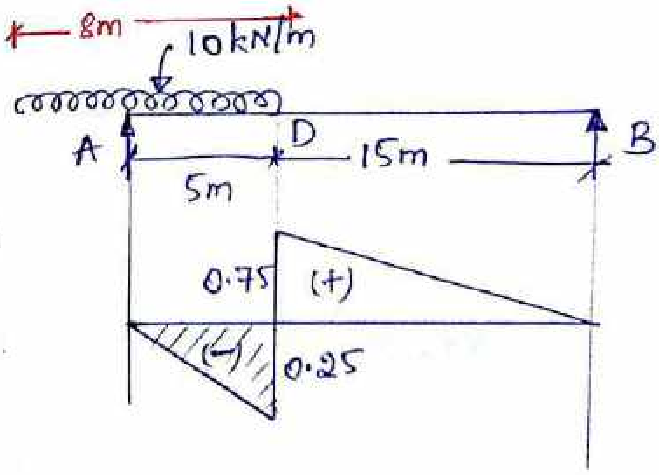
$= \frac{0.75}{15} \times 7 = \underline{\underline{0.35}}$

$\therefore$ Maximum +ve S.F = Intensity of load × area of ILD under load.

$= 10 \times \left[\left(\tfrac{1}{2} \times 8 \times (0.75 - 0.35) \right) + (0.35 \times 8) \right]$

$= \underline{44 \text{ kN}}$

c) Maximum -ve S.F : Max. -ve S.F due to an UDL occurs when the ~~load of is at~~ head of the load is at D as it traverses from left to right



$\therefore$ Max -ve S.F $= 10 \times (\tfrac{1}{2} \times 5 \times 0.25)$

$= \underline{\underline{6.25 \text{ kN}}}$

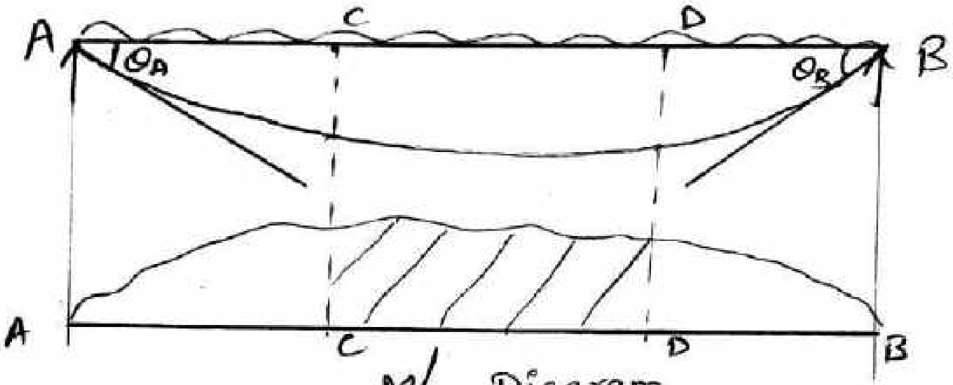
MODULE-03

DEFLECTION OF BEAMS

Moment -Area Method

Mohr's I- Theorem :-

It is the theorem of slope which states that "Difference of slope at any two points in the loaded elastic beam is equal to the area of M/EI diagram between those two points."



M/EI Diagram

$\theta_A - \theta_B$ = Area of M/EI diagram between A & B.

$\theta_A - \theta_C$ = Area of M/EI diagram between A & C

Mohr's II - Theorem :-

It is the theorem of deflection which states that "the Vertical distance of displaced position of any point in the loaded elastic beam from the tangent drawn at another point is equal to the moment of M/EI diagram between those two points about the point at which deflection is required."

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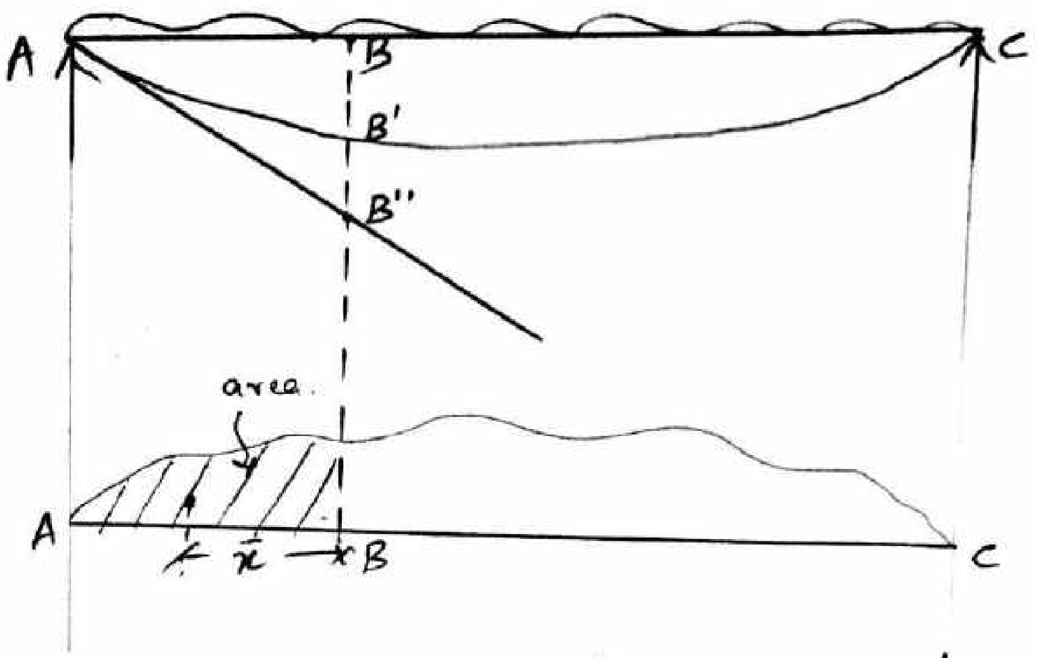
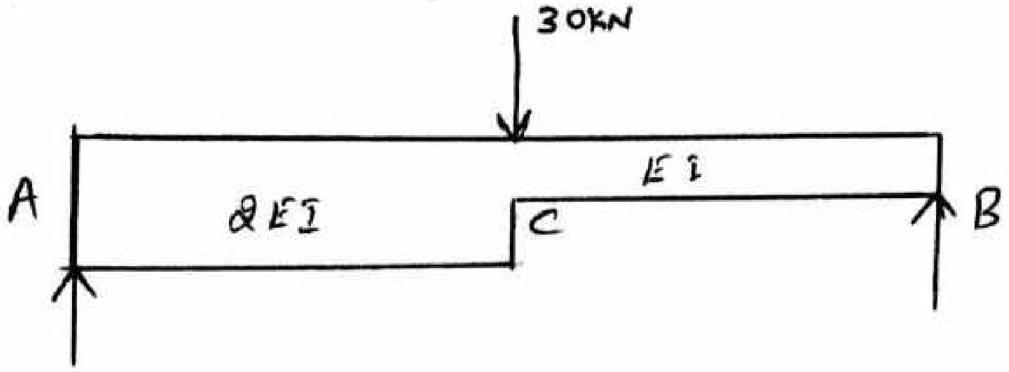


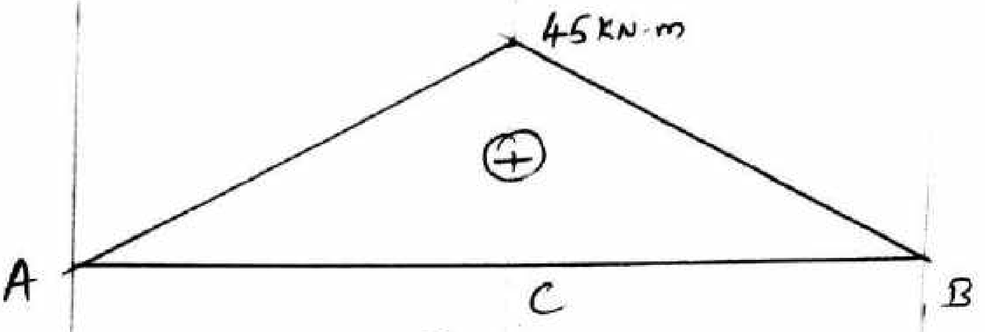
$B'B''$ = Moment of area of M/EI diagram between A & B about B.

$$\boxed{B'B'' = A \cdot \bar{x}}$$

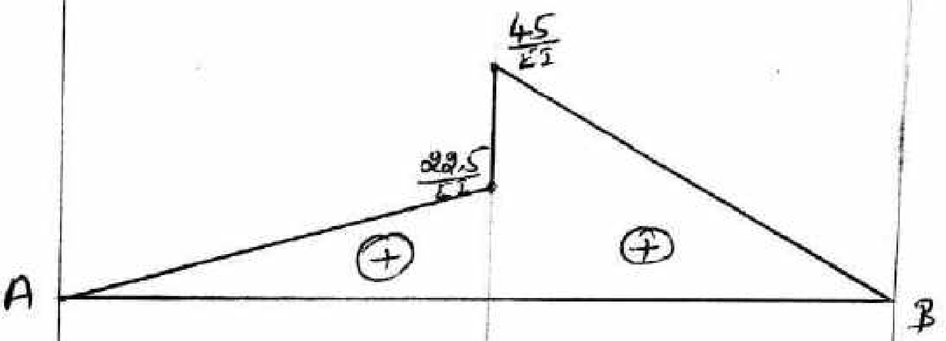
Note :-

* Positive Area is positive and negative area is negative. in M/EI diagram.

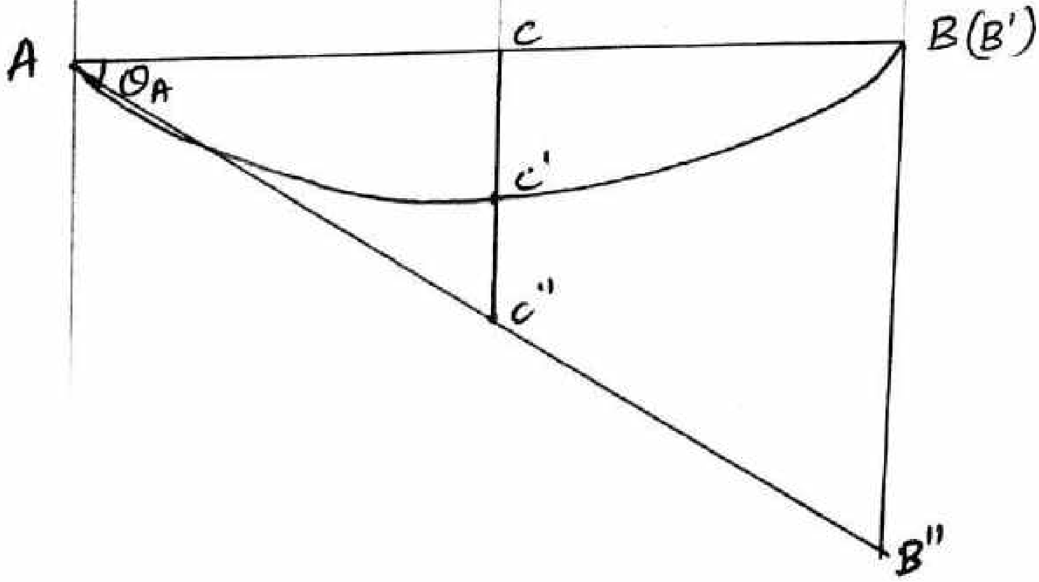




BMD



M/EI Diagram



Problems On Simply Supported Beam

1) Determine the slope at supports and deflection under the load for the beam shown in fig using moment-area Method.

→ **Support Reaction:-**

By symmetry, $R_A = R_B = 15KN$.

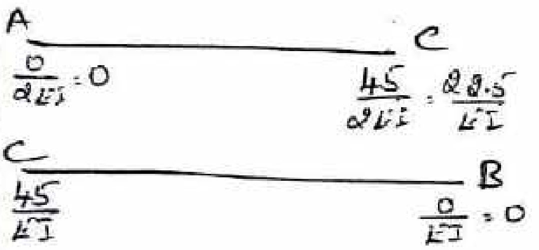
BMD:-

$BM_A = 0$

$BM_c = \frac{Wab}{L} = \frac{30\times3\times3}{6} = 45KN\cdot m$

$BM_B = 0$

M/EI Diagram :-



Slope :-

From Mohr's II - Theorem

$B'B''$ = Moment of area of M/EI diagram b/w A & B about B.

$B'B'' = \left(\frac{1}{2}\times3\times\frac{22.5}{EI}\right)\left[\left(\frac{1}{3}\times3\right)+3\right] + \left(\frac{1}{2}\times3\times\frac{45}{EI}\right)\left(\frac{2}{3}\times3\right)$

$\boxed{B'B'' = \frac{270}{EI}}$

From Δ^{le} $AB'B''$,

$\theta_A = \frac{B'B''}{AB}$

$\theta_A = \frac{270/EI}{6} = \frac{45}{EI}$

From Mohr's I - Theorem,

$(\theta_A - \theta_B)$ = Area of M/EI diagram b/w A & B

$\theta_A - \theta_B = \left(\frac{1}{2}\times3\times\frac{22.5}{EI}\right) + \left(\frac{1}{2}\times3\times\frac{45}{EI}\right)$

$\boxed{\theta_B = \frac{56.25}{EI}}$

Deflection (CC') :-

$C'C''$ = Moment of area of M/EI diagram b/w A & C about C

$C'C'' = \left(\frac{1}{2}\times3\times\frac{22.5}{EI}\right)\left(\frac{1}{3}\times3\right)$

$\boxed{C'C'' = \frac{33.75}{EI}}$

From Δ^{le}, ACC'',

$\theta_A = \frac{CC''}{AC}$

$CC'' = \frac{45}{EI}\times3$

$\boxed{CC'' = \frac{135}{EI}}$

$CC' = CC'' - C'C''$

$= \frac{135}{EI} - \frac{33.75}{EI}$

$\boxed{CC' = \frac{101.25}{EI}}$

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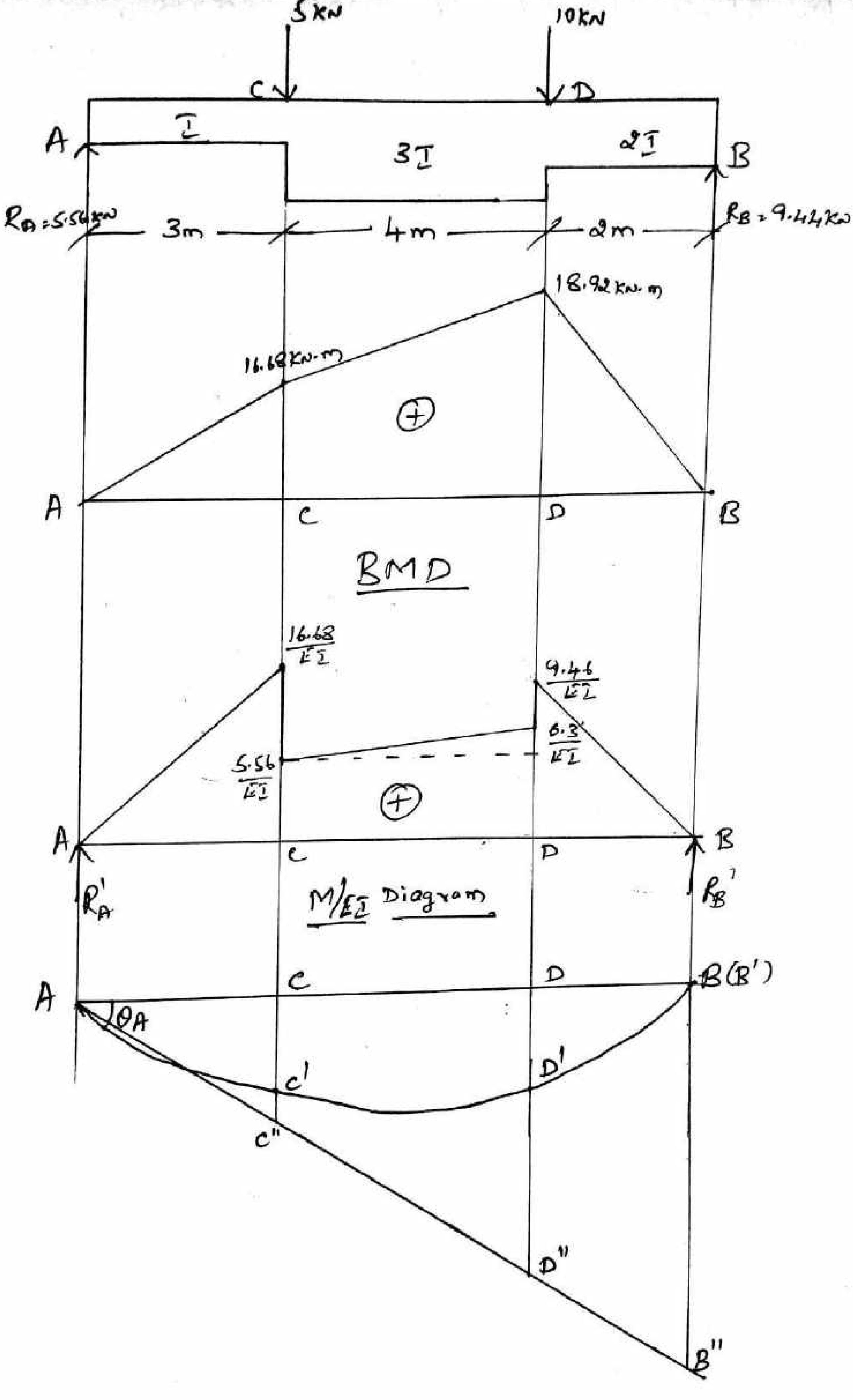
5KN
10KN
C
D
A
I
3I
2I
B
RA=5.56KN
3m
4m
2m
RB=9.44KN
18.92 KN.m
16.68 KN.m
A
C
D
B
BMD
16.68/EI
9.46/EI
6.3/EI
5.56/EI
A
C
D
B
R'A
R'B
M/EI Diagram
A
C
D
B(B')
θA
C'
D'
C''
D''
B''

2) Find the slope at 'A' & deflection at the load Points in terms of flexural Rigidity using moment-area method.

→ Support Reaction:-

$\Sigma M_A = 0, (5\times3) + (10\times7) - (R_B\times9) = 0$

$\boxed{R_B = 9.44 KN}$

$\Sigma F_y = 0, \quad R_A - 5 - 10 + 9.44 = 0$

$\boxed{R_A = 5.56 KN}$

BMD:-

$BM_A = 0$

$BM_C = (5.56\times3) = 16.68 KN\cdot m$

$BM_D = (5.56\times7) - (5\times4) = 18.92 KN\cdot m$

$BM_B = 0.$

M/EI Diagram:-

A ———— C

$0 \qquad \frac{16.68}{EI}$

C ———— D

$\frac{16.68}{3EI} = \frac{5.56}{EI} \qquad \frac{18.92}{EI} = \frac{6.3}{EI}$

D ———— B

$\frac{18.92}{2EI} = \frac{9.46}{EI} \qquad \frac{0}{EI} = 0$

Slope at A (θ_A):-

From Mohr's II - Theorem:-

$B'B''$ = Moment of area of M/EI diagram b/w A & B about B

$B'B'' = \left(\frac{1}{2}\times3\times\frac{16.68}{EI}\right)\left[\left(\frac{1}{3}\times3\right)+6\right]$

$+ \left(4\times\frac{5.56}{EI}\right)\left(\frac{4}{2}+2\right)$

$+ \left(\frac{1}{2}\times4\times\frac{0.74}{EI}\right)\left(\left[\frac{1}{3}\times4\right)+2\right]$

$+ \left(\frac{1}{2}\times2\times\frac{9.46}{EI}\right)\left(\frac{2}{3}\times2\right)$

$\boxed{B'B'' = \frac{281.64}{EI}}$

From $A'AB'B''$,

$\theta_A = \frac{B'B''}{AB} = \frac{\frac{281.64}{EI}}{9}$

$\boxed{\theta_A = \frac{31.30}{EI}}$

Deflection $(CC' \& DD')$:-

From Mohr's II Theorem

$C'C'' = \left(\frac{1}{2}\times3\times\frac{16.68}{EI}\right)\left(\frac{1}{3}\times3\right)$

$\boxed{C'C'' = \frac{25.02}{EI}}$

$\theta_A = \frac{CC''}{AC}$

$CC'' = \frac{31.30}{EI}\times3$

$\boxed{CC'' = \frac{93.87}{EI}}$

$CC' = CC'' - C'C''$

$= \frac{93.87}{EI} - \frac{25.02}{EI}$

$\boxed{CC' = \frac{68.85}{EI}}$

From Mohr's II Theorem:-

$D'D'' = \left(\frac{1}{2}\times3\times\frac{16.68}{EI}\right)\left[\frac{1}{3}\times3+4\right]$

$+ \left(\frac{1}{2}\times4\times\frac{0.74}{EI}\right)\left(\frac{1}{3}\times4\right) + \left(4\times\frac{5.56}{EI}\right)\left(\frac{4}{2}\right)$

$\boxed{D'D'' = \frac{171.55}{EI}}$

$\theta_A = \frac{DD''}{AD} = \frac{\cancel{31.29}}{\cancel{EI}} \qquad DD'' = \frac{31.3}{EI}\times7$

$\cancel{\theta_A} = \boxed{DD'' = \frac{219.03}{EI}}$

$DD' = DD'' - D'D'' = \frac{47.48}{EI}.$

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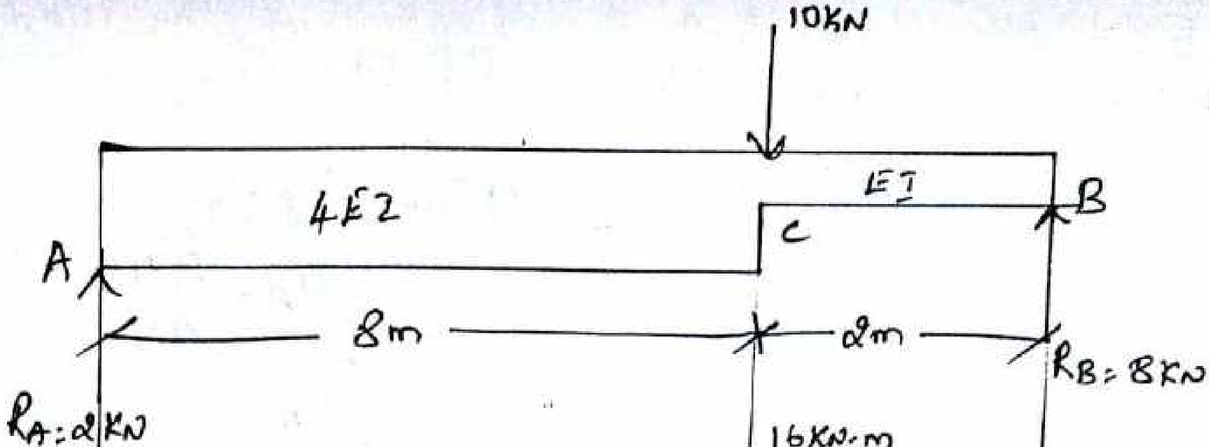
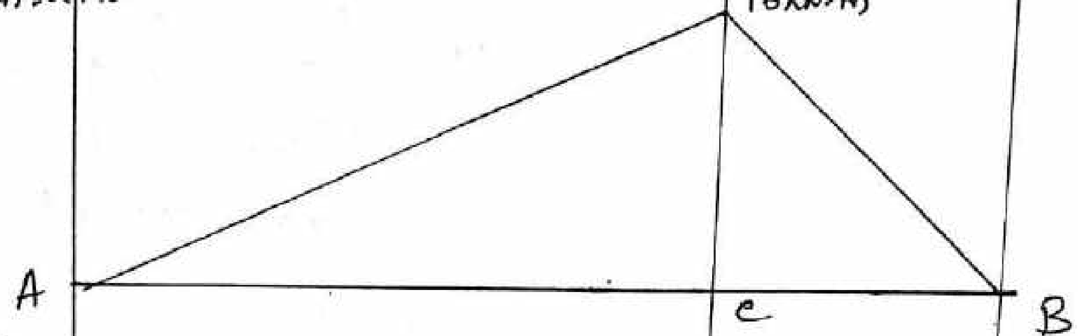
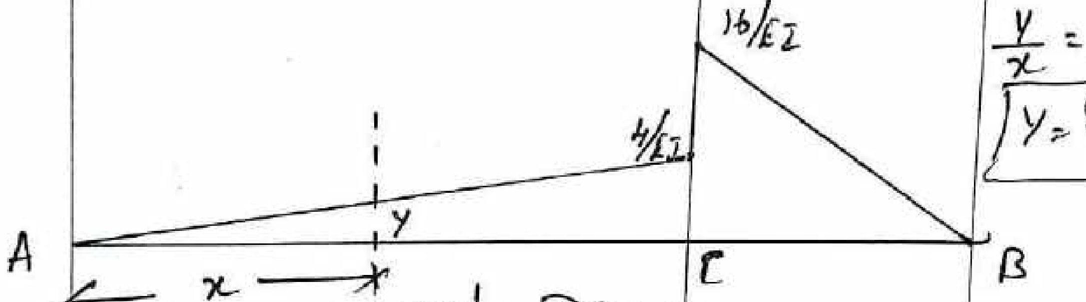




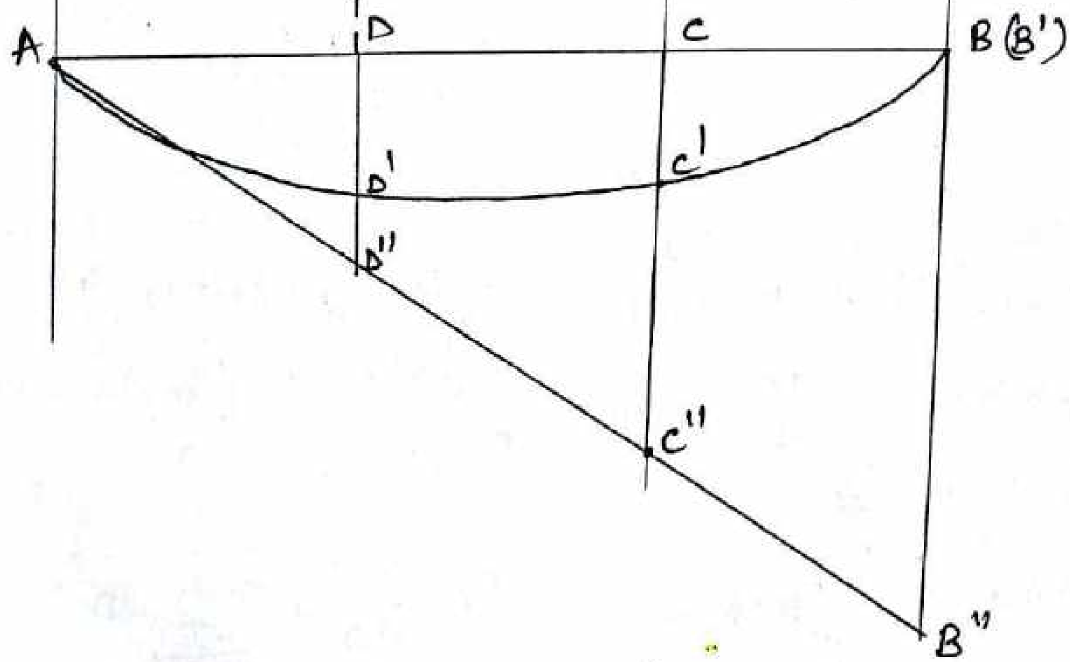
BMD



$$\frac{y}{x} = \frac{4/EI}{8}$$

$$\boxed{y = \frac{0.5x}{EI}}$$

M/EI Diagram



3) Using moment area Method, Calculate slope & maximum deflection & its location for the beam shown in figure.

→ Support Reaction:-

$\Sigma M_A = 0, (10 \times 8) - R_B \times 10 = 0$

$\boxed{R_B = 8 KN}$

$R_A + 8 - 10 = 0$

$\boxed{R_A = 2 KN}$

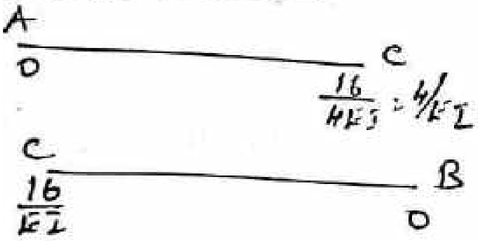
BMD:-

$BM_A = 0$

$BM_C = 2 \times 8 = 16 KN\text{-}m$

$BM_B : 0$

M/EI Diagram:-



Slope:-

From Mohr's -II- Theorem:-

B'B'' = Moment of area of M/EI diagram b/w A&B about B.

$B'B'' = \left(\frac{1}{2} \times 8 \times 4/EI\right)\left[\frac{1}{3} \times 8 + 2\right] + \left(\frac{1}{2} \times 2 \times 16/EI\right)\left(\frac{2}{3} \times 2\right)$

$\boxed{B'B'' = \frac{96}{EI}}$

From Δ^{le} AB'B'', $\theta_A = \frac{B'B''}{AB}$

$\theta_A = \frac{96/EI}{10}$

$\boxed{\theta_A = \frac{9.6}{EI}}$

From Mohr's -I- Theorem

$\theta_A - \theta_B$ = Area of M/EI diagram b/w A & B.

$\theta_A - \theta_B = \left(\frac{1}{2} \times 8 \times \frac{4}{EI}\right) + \left(\frac{1}{2} \times 2 \times \frac{16}{EI}\right)$

$\boxed{\theta_B = \frac{22.4}{EI}}$

Maximum Deflection:- (DD')

Let us assume that max deflection occurs at a distance 'x' from A, i.e, at Point 'D'.

$\theta_A - \theta_D = \left(\frac{1}{2} \times x \times y\right)$

$\theta_A - \frac{1}{2} \times x \times \frac{0.5x}{EI} = \theta_D$

$\frac{9.6}{EI} - \frac{1}{2} \times x \times \frac{0.5x}{EI} = 0 \quad (\because \theta_D = 0)$

$\boxed{x = 6.19 m}$

$\therefore$ D'D'' = moment of area of M/EI diagram b/w A&D about D.

$D'D'' = \left(\frac{1}{2} \times 6.19 \times \frac{0.5 \times 6.19}{EI}\right)\left(\frac{1}{3} \times 6.19\right)$

$\boxed{D'D'' = \frac{19.76}{EI}}$

$\theta_A = \frac{DD''}{AD}$

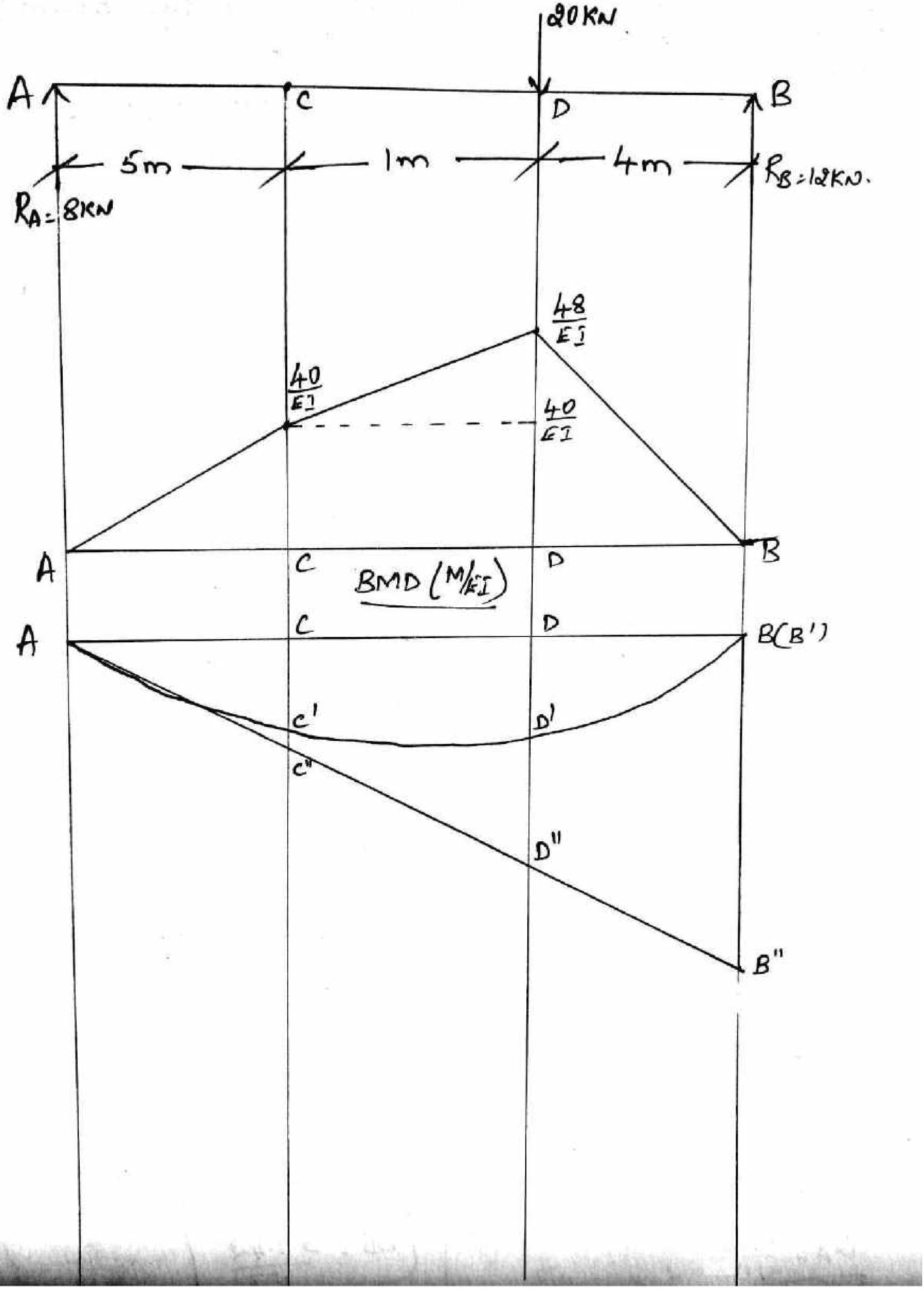
$DD'' = \frac{9.6}{EI} \times 6.19$

$\boxed{DD'' = \frac{59.42}{EI}}$

$DD' = DD'' - D'D''$

$= \frac{59.42}{EI} - \frac{19.76}{EI}$

$\boxed{DD' = \frac{39.67}{EI}}$


20KN
A
C
D
B
5m
1m
4m
RA = 8KN
RB = 12KN.
48/EI
40/EI
40/EI
BMD (M/EI)
B(B')
C'
D'
C''
D''
B''

4) Find slope at A, slope at D & deflection at D for the beam shown in fig. using moment area method

→ Support Reaction:-

$\Sigma M_A = 0,$

$(20\times 6) - R_B \times 10 = 0$

$\boxed{R_B = 12\,kN}$

$\Sigma F_y = 0,$

$R_A - 20 + 12 = 0$

$\boxed{R_A = 8\,kN}$

BMD:-

$BM_A = 0,\ BM_c = (8\times 5) = 40\,kN\cdot m$

$BM_D = (8\times 6) = 48\,kN\cdot m,\ BM_B = 0.$

Slope:-

From Mohr's II Theorem,

B'B'' = moment of area of M/EI diagram b/w ~~about~~ A & B about B.

$B'B'' = \left(\frac{1}{2}\times 5\times\frac{40}{EI}\right)\left[\left(\frac{1}{3}\times 5\right)+5\right] + \left(\frac{1}{2}\times 1\times\frac{8}{EI}\right)\left[\left(\frac{1}{3}\times 3\right)+4\right] + \left[1\times\frac{40}{EI}\right]\left[\frac{1}{2}+4\right] + \left(\frac{1}{2}\times 4\times\frac{48}{EI}\right)\left(\frac{2}{3}\times 4\right)$

$\boxed{B'B'' = \frac{1120}{EI}}$

$\theta_A = \frac{B'B''}{AB}$

$\theta_A = \frac{1120/EI}{10}$

$\boxed{\theta_A = \frac{112}{EI}}$

$\theta_A - \theta_D$ = area of M/EI diagram between A & D

$\theta_A - \theta_D = \left(\frac{1}{2}\times 5\times\frac{40}{EI}\right) + \left(\frac{1}{2}\times 1\times\frac{8}{EI}\right) + \left(1\times\frac{40}{EI}\right)$

$\boxed{\theta_D = \frac{-32}{EI}}$

Deflection:-(DD')

From Mohr's II Theorem,

D'D'': moment of area of M/EI diagram b/w A & D about D.

$D'D'' = \left(\frac{1}{2}\times 5\times\frac{40}{EI}\right)\left[\left(\frac{1}{3}\times 5\right)+1\right] + \left(\frac{1}{2}\times 1\times\frac{8}{EI}\right)\left[\frac{1}{3}\times 1\right] + \left(\frac{40}{EI}\times 1\right)\left(\frac{1}{2}\right)$

$\boxed{D'D'' = \frac{288}{EI}}$

$\theta_A = \frac{DD''}{AD}$

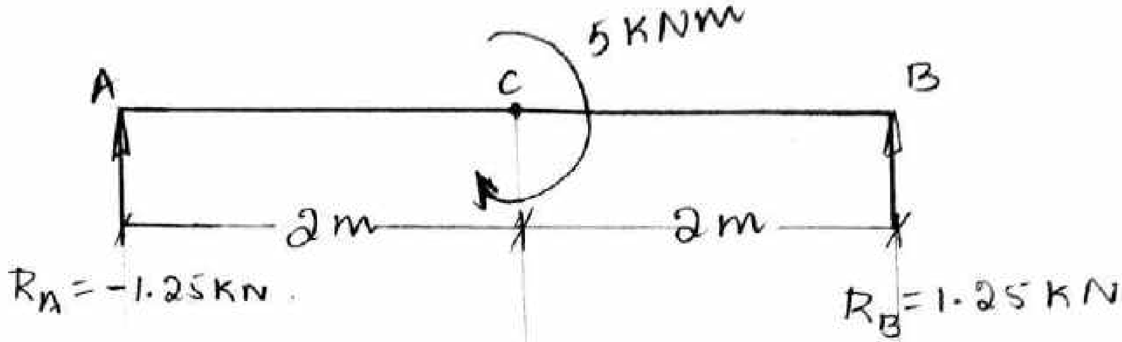
$DD'' = \frac{112}{EI}\times 6$

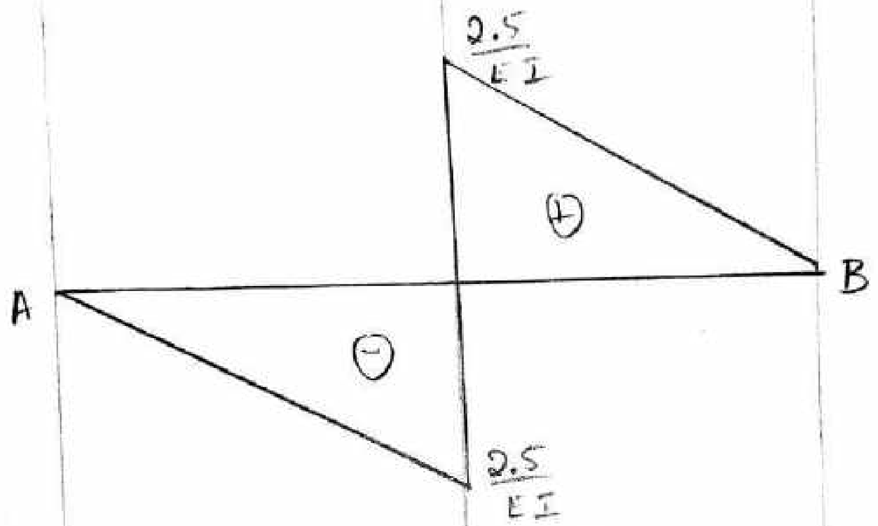
$\boxed{DD'' = \frac{672}{EI}}$

$DD' = DD'' - D'D''$

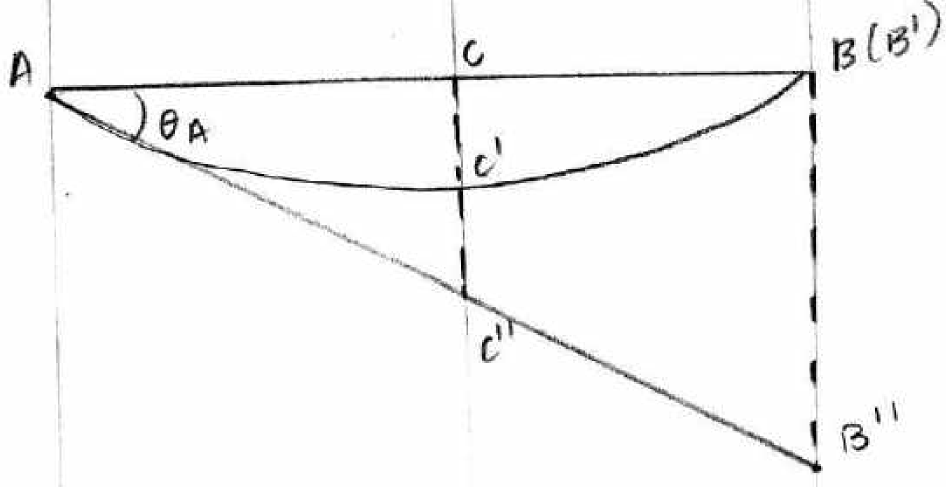
$= \frac{672}{EI} - \frac{288}{EI}$

$\boxed{DD' = \frac{384}{EI}}$


5 KNm
A
C
B
2m
2m
$R_A = -1.25$ KN
$R_B = 1.25$ KN


$\frac{2.5}{EI}$
+
A
B
−
$\frac{2.5}{EI}$

BMD [M/EI]


A
C
B(B')
θ_A
c'
c''
B''

5) A simply supported beam is subjected to a couple at the center. Find the slope at the ends and deflection at the center :-

Sol^n:- → Support reaction:-

$\Sigma M_A = 0$

$5 - (R_B \times 4) = 0 \Rightarrow \boxed{R_B = 1.25\,KN}$

$\Sigma F_y = 0$

$R_A + R_B = 0 \Rightarrow \boxed{R_A = -1.25\,KN}$

→ BMD:-

$BM_A = 0$

$BM_{C\,without\,couple} = (-1.25 \times 2) = -2.5\,KNm.$

$BM_{C\,with\,couple} = -2.5 + 5 = 2.5\,KNm.$

$BM_B = 0$

Slope:-

From Mohr's 2nd theorem,

$B'B''$ = Moment of area of M/EI diagram b/w A & B about B.

$$B'B'' = -\left[\frac{1}{2} \times 2 \times \frac{2.5}{EI}\right]\left[\frac{1}{3} \times 2 + 2\right] + \left[\frac{1}{2} \times 2 \times 2.5\right]\left[2/3 \times 2\right]$$

$$\boxed{B'B'' = \frac{-3.33}{EI}}$$

$$\theta_A = \frac{B'B''}{AB} = \frac{-3.33}{4EI}$$

$$\boxed{\theta_A = \frac{-0.834}{EI}}$$

From Mohr's 1st theorem,

$\theta_A - \theta_B$ = Area of $\frac{M}{EI}$ diagram b/w A and B.

$$\theta_A - \theta_B = -\left[\frac{1}{2} \times 2 \times 2.5\right] + \left[\frac{1}{2} \times 2 \times 2.5\right]$$

$\theta_A = \theta_B$

$$\boxed{\theta_B = \frac{-0.834}{EI}}$$

Deflection at center:-

$C'C''$ = Moment of area of M/EI diagram b/w A & C about C.

$$C'C'' = -\left[\frac{1}{2} \times 2 \times \frac{2.5}{EI}\right]\left[\frac{1}{3} \times 2\right]$$

$$\boxed{C'C'' = \frac{-1.67}{EI}}$$

From $\Delta ACC''$,

$$\theta_A = \frac{CC''}{AC}$$

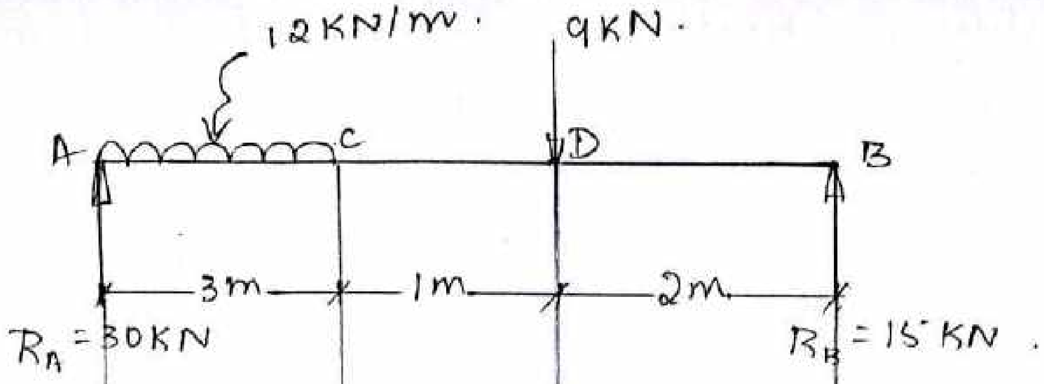
$$CC'' = AC(\theta_A) = 2\left[\frac{-0.834}{EI}\right]$$

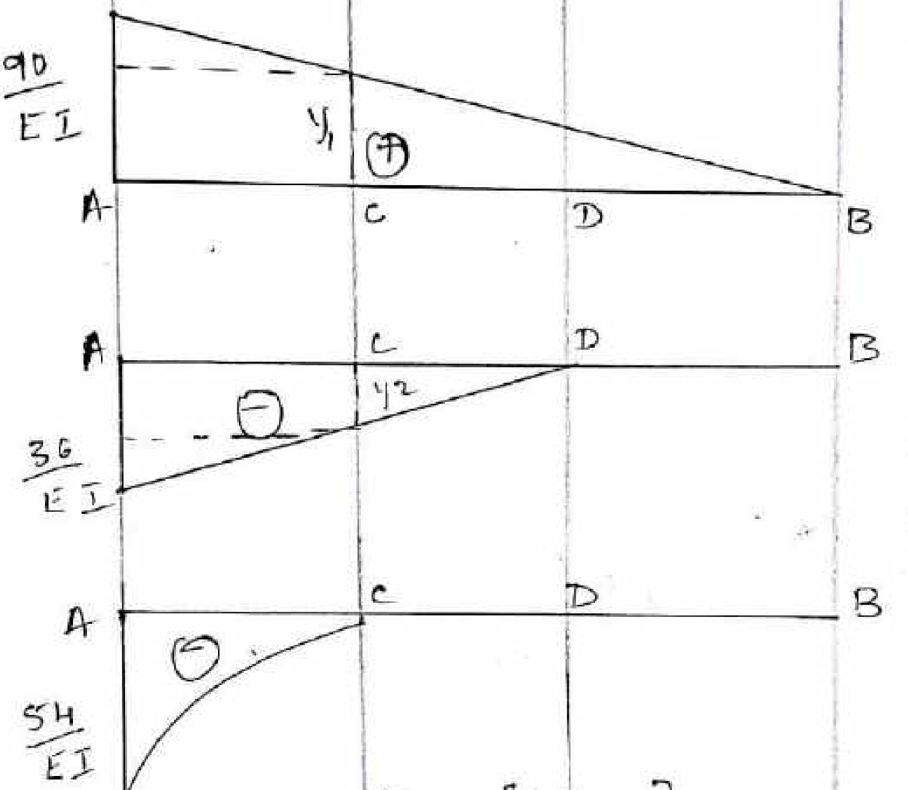
$$CC'' = \frac{-1.668}{EI}$$

$$CC' = CC'' - C'C'' = \frac{-1.668}{EI} - \left[\frac{-1.67}{EI}\right]$$

$$\boxed{CC' = 0.002}$$

$$\boxed{CC' \simeq 0}$$





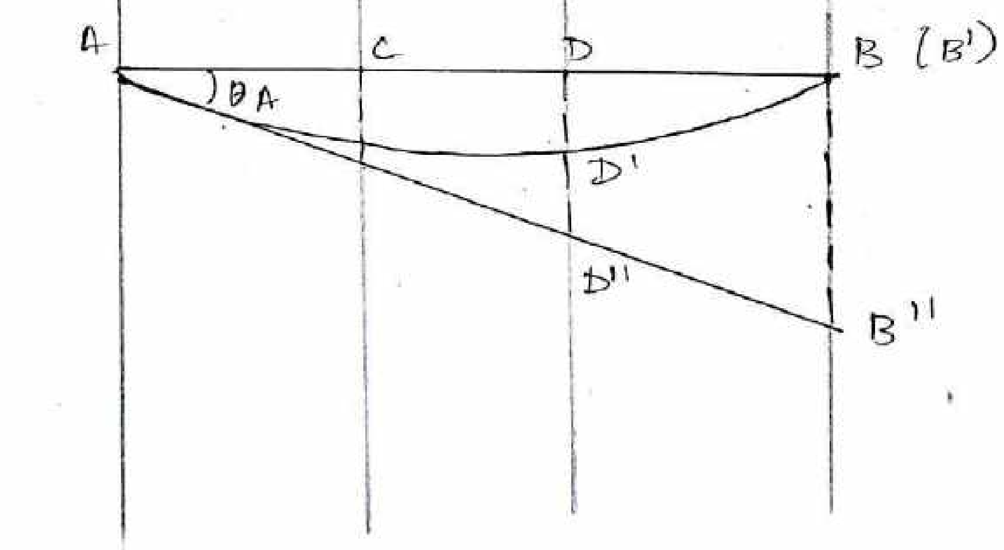
BMD [M/EI]

$$\frac{y_1}{3} = \frac{90}{6EI}$$

$$\boxed{y_1 = \frac{45}{EI}}$$

$$\frac{y_2}{1} = \frac{36}{4EI}$$

$$\boxed{y_2 = 9/EI}$$



6). Determine the slope at A and deflection at C for the beam shown in the fig. using moment-Area method :-

<u>Support reactions</u> :-

$\Sigma M_A = 0$

$(12\times3)(3/2) + (9\times4) - (R_B\times6) = 0$

$\boxed{R_B = 15\ KN}$

$\Sigma F_y = 0$.

$R_A - (12\times3) - 9 + R_B = 0$.

$\boxed{R_A = 30\ KN}$

<u>BMD</u> :-

BM_A due to $R_B = (15\times6) = 90\ KNm$

BM_A due to $9KN = (9\times4) = 36\ KNm$

BM_A due to UDL $= (12\times3)(3/2) = 54\ KNm$

<u>Slope</u> :-

By Mohr's 2nd theorem,

$B'B''$ = Moment of area of M/EI diagram b/w A & B about B.

$$B'B'' = \left[\frac{1}{2}\times6\times\frac{90}{EI}\right]\left[\frac{2}{3}\times6\right] - \left[\frac{1}{2}\times4\times\frac{36}{EI}\right]\left[2+\frac{2}{3}\times4\right] - \left[\frac{1}{3}\times3\times\frac{54}{EI}\right]\left[3+\frac{3}{4}(3)\right]$$

$$\therefore \boxed{B'B'' = \frac{460.5}{EI}}$$

$$\theta_A = \frac{B'B''}{AB} = \frac{460.5}{6EI}$$

$$\boxed{\theta_A = \frac{76.75}{EI}}$$

From Mohr's 1st theorem,

$\theta_A - \theta_B$ = Area of $\frac{M}{EI}$ diagram b/w A and B.

$$\theta_A - \theta_B = \left[\frac{1}{2}\times6\times\frac{90}{EI}\right] - \left[\frac{1}{2}\times4\times\frac{36}{EI}\right] - \left[\frac{1}{3}\times3\times54/EI\right]$$

$$\boxed{\theta_B = \frac{67.25}{EI}}$$

<u>Deflection :- (CC')</u> :-

From Mohr's 2nd theorem,

$C'C''$ = Moment of area of M/EI diagram b/w A & C about C.

$$C'C'' = \left[\frac{1}{2}\times3\times\frac{45}{EI}\right]\left[\frac{2}{3}\times3\right] + \left[\frac{45}{EI}\times3\right]\left[\frac{3}{2}\right] - \left[\frac{1}{2}\times3\times\left(\frac{36-9}{EI}\right)\right]\left[\frac{2}{3}\times3\right] - \left[\frac{9}{EI}\times3\right]\left[3/2\right] - \left[\frac{1}{3}\times3\times\frac{54}{EI}\right]\left[\frac{3}{4}\times3\right]$$

$$\boxed{C'C'' = \frac{94.5}{EI}}$$

From $\Delta ACC''$, $\theta_A = \frac{CC''}{AC}$.

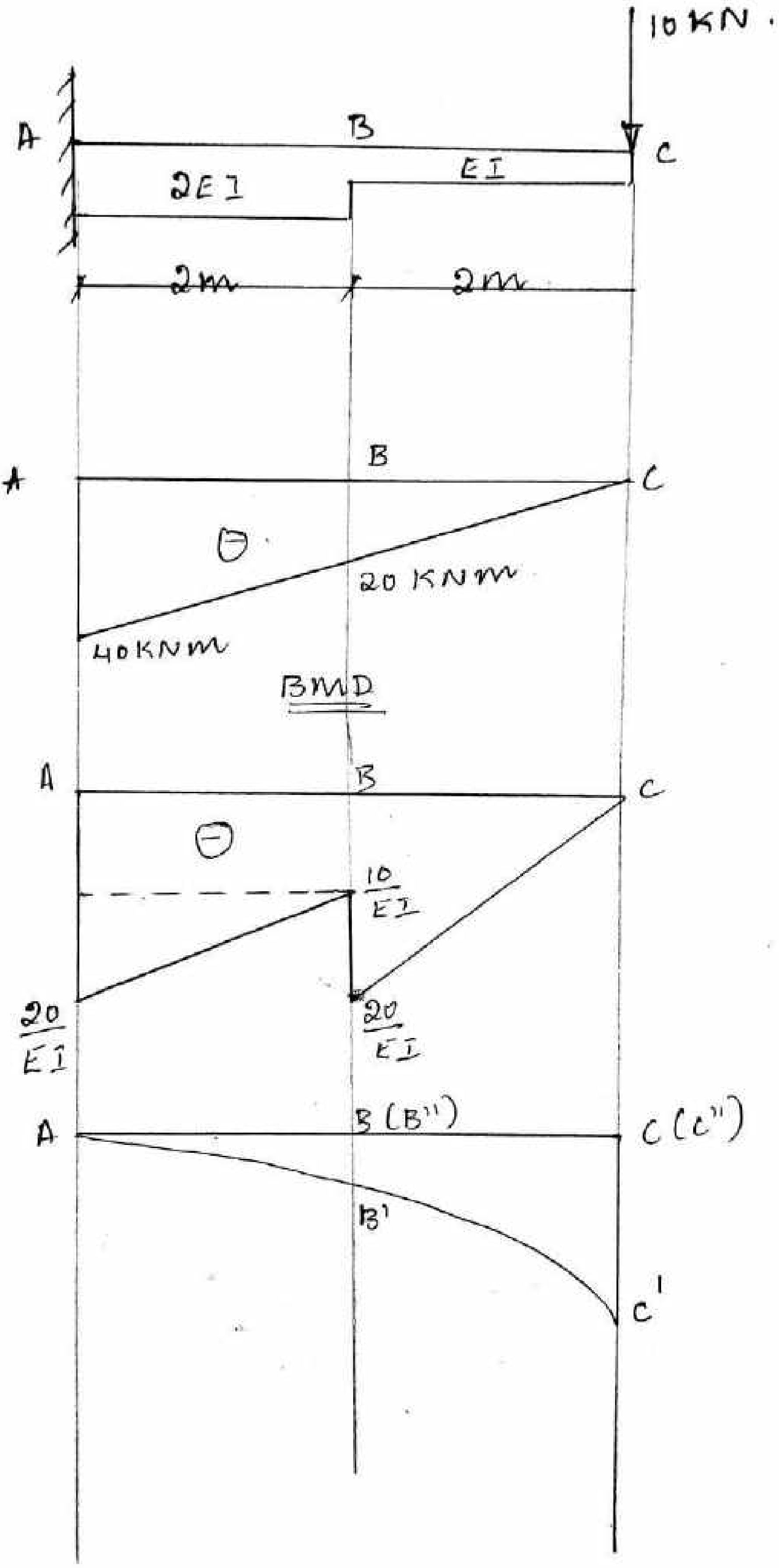
$$CC'' = AC(\theta_A) = 3\left[\frac{76.75}{EI}\right]$$

$$\boxed{CC'' = \frac{230.25}{EI}}$$

$$\therefore CC' = CC'' - C'C'' = \frac{230.25}{EI} - \frac{94.5}{EI}.$$

$$\boxed{CC' = \frac{135.75}{EI}}$$

Problems on Cantilever Beam:-



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8). Find the maximum deflection and slope at free end in the cantilever beam shown in the fig:-

Note:- In cantilever beam, slope at fixed end is 0.
∴ The tangent drawn from the fixed end coincides with the real beam.

→ BMD:-

$BM_A = (10 \times 4) = 40\ KNm$

$BM_B = (10 \times 2) = 20\ KNm$

$BM_c = 0$.

M/EI diagram:-

A $\frac{40}{2EI} = \frac{20}{EI}$ ———— $\frac{20}{2EI} = \frac{10}{EI}$ B

B $20/EI$ ———— 0 C

Slope:-

Slope at c:- From Mohr's 1st theorem,

$\theta_A - \theta_C$ = Area of M/EI diagram b/w A and c.

$$\theta_A - \theta_c = -\left[\frac{10}{EI} \times 2\right] - \left[\frac{1}{2} \times 2 \times \frac{10}{EI}\right] - \left[\frac{1}{2} \times 2 \times \frac{20}{EI}\right]$$

$$0 - \theta_c = \frac{-50}{EI}$$

$$\boxed{\theta_c = \frac{50}{EI}}$$

Maximum deflection at c:-

From Mohr's 2nd theorem,

$cc' = c'c''$ = Moment of area of M/EI diagram b/w A & C about c.

$$cc' = c'c'' = -\left[\frac{1}{2} \times 2 \times \frac{10}{EI}\right]\left[2 + \frac{2}{3} \times 2\right] - \left[\frac{10}{EI} \times 2\right]\left[2 + 2/2\right] - \left[\frac{1}{2} \times 2 \times \frac{20}{EI}\right]\left[2/3 \times 2\right]$$

$$\boxed{cc' = -\frac{120}{EI}}$$

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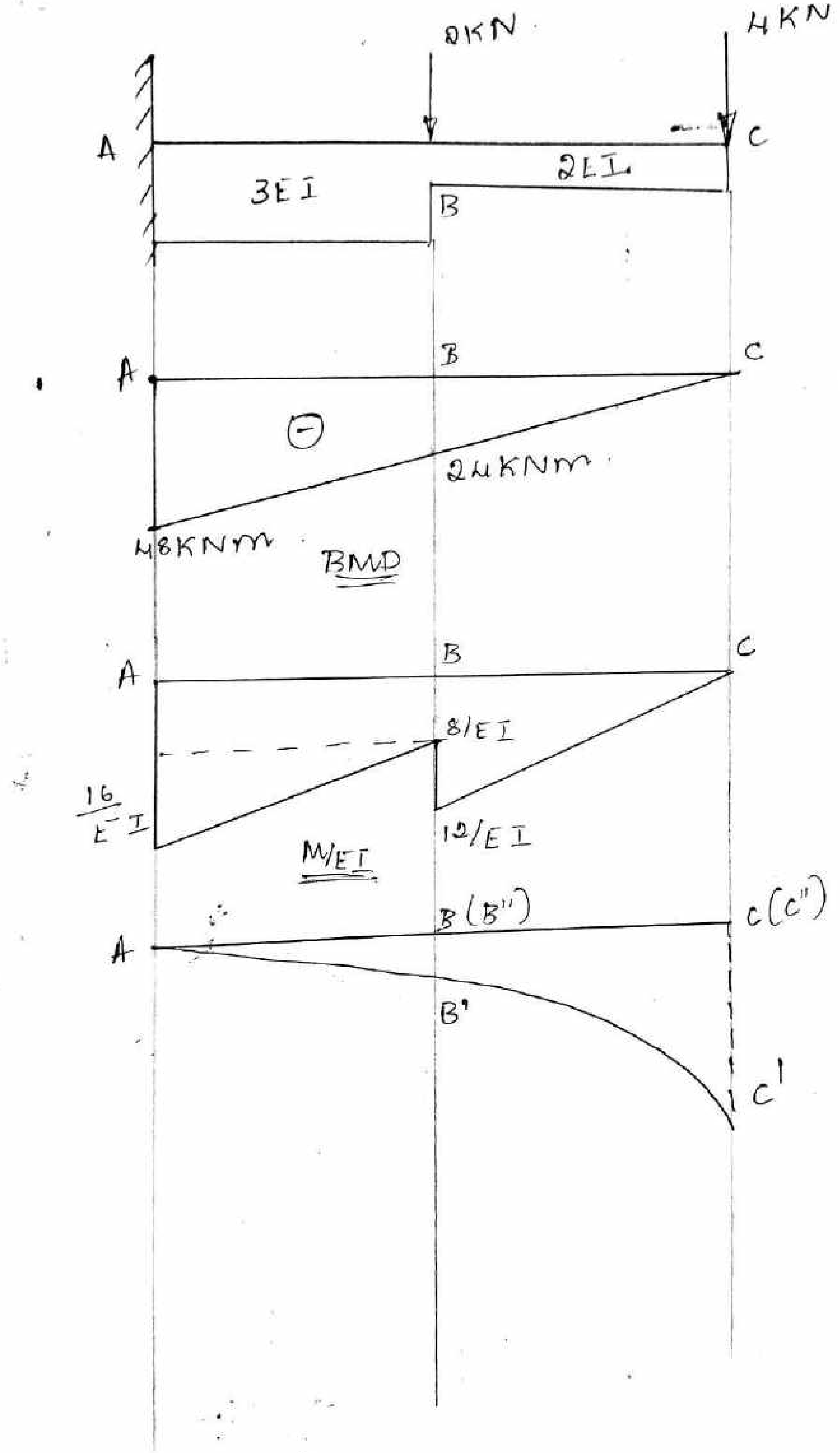

2KN
4KN
A
C
3EI
2EI
B
A
B
C
24KNm
48KNm
BMD
A
B
C
8/EI
16/EI
12/EI
M/EI
B(B'')
C(C'')
A
B'
C'

9) Find deflection and slope at B in a cantilever beam loaded as shown in fig. below :-

BMD :-

$BM_A = (4 \times 10) + (2 \times 4) = 48 \text{ kNm}$

$BM_B = (4 \times 6) = 24 \text{ kNm}$

$BM_C = 0$

M/EI diagram :-

A $\frac{48}{3EI} = \frac{16}{EI}$ $\frac{24}{3EI} = \frac{8}{EI}$. B

B $\frac{24}{2EI} = \frac{12}{EI}$ 0 C

Deflection :-

From Mohr's 2nd theorem,

$BB' = B'B''$ = Moment of Area of M/EI diagram b/w A & B about B

$$BB' = B'B'' = -\left[\frac{1}{2} \times 4 \times \left(\frac{16-8}{EI}\right)\right]\left[\frac{2}{3} \times 4\right] - \left[\frac{8}{EI} \times 4\right]\left[\frac{4}{2}\right]$$

$$\boxed{BB' = -\frac{106.67}{EI}}$$

Slope at B :-

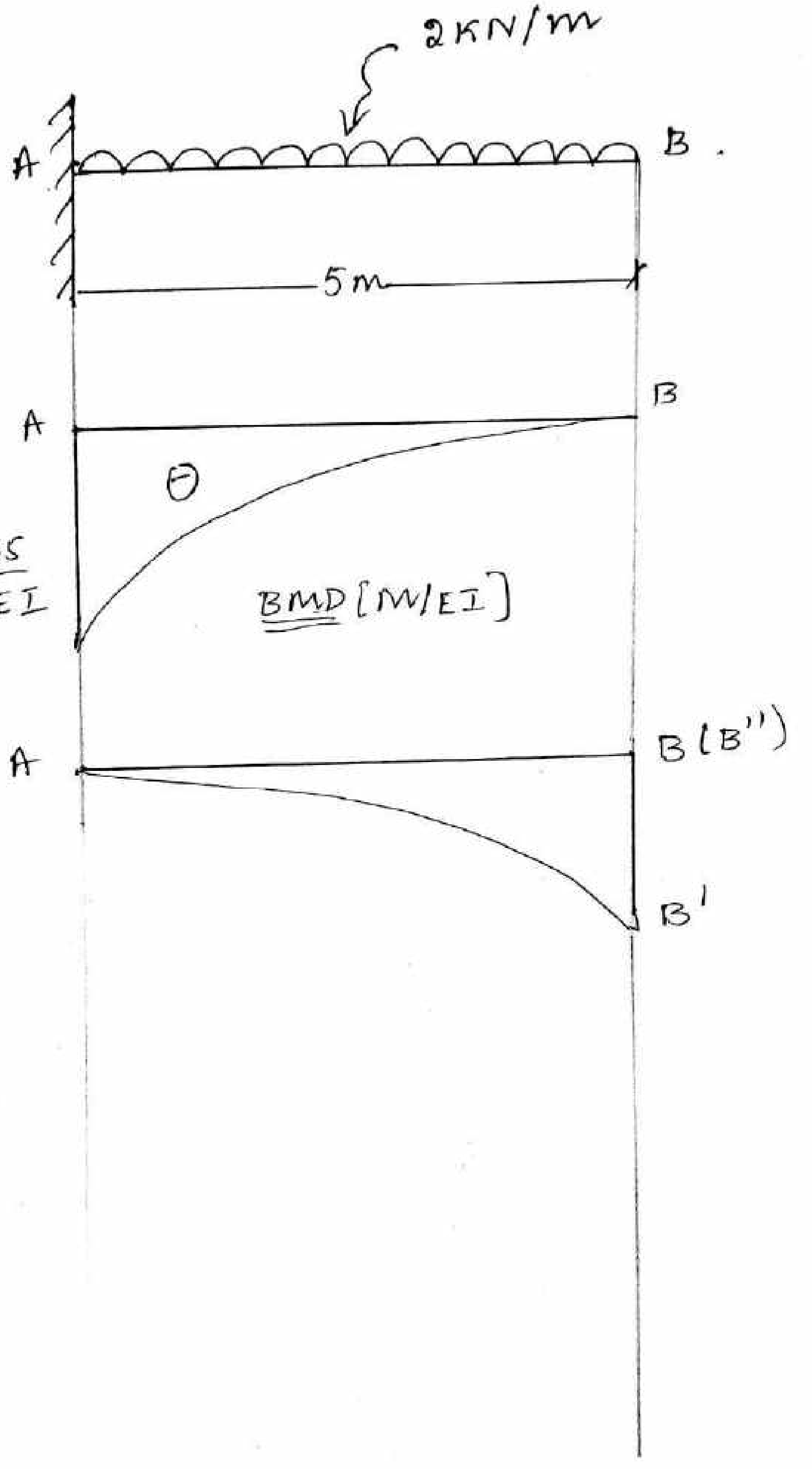
From Mohr's Ist theorem,

$\theta_A - \theta_B$ = Area of M/EI diagram b/w A & B.

$$= -\left[\frac{1}{2} \times 4 \times \left[\frac{16-8}{EI}\right]\right] - \left[\frac{8}{EI} \times 4\right]$$

$$0 - \theta_B = -\frac{48}{EI}.$$

$$\boxed{\theta_B = \frac{48}{EI}}$$


2KN/m
A
B.
5m
A
B
25/EI
BMD [M/EI]
A
B (B'')
B'

10). Find the max deflection and slope at free end for the cantilever beam shown in fig below:-

Sol^n: BMD :-

$BM_A = (2\times5)(5/2) = 25\ KNm$.

Slope:-

From Mohr's 1st theorem,

$\theta_A - \theta_B =$ Area of M/EI diagram b/w A and B.

$0 - \theta_B = -\left[\frac{1}{3}\times5\times\frac{25}{EI}\right]$

$$\boxed{\theta_B = \frac{41.66}{EI}}$$

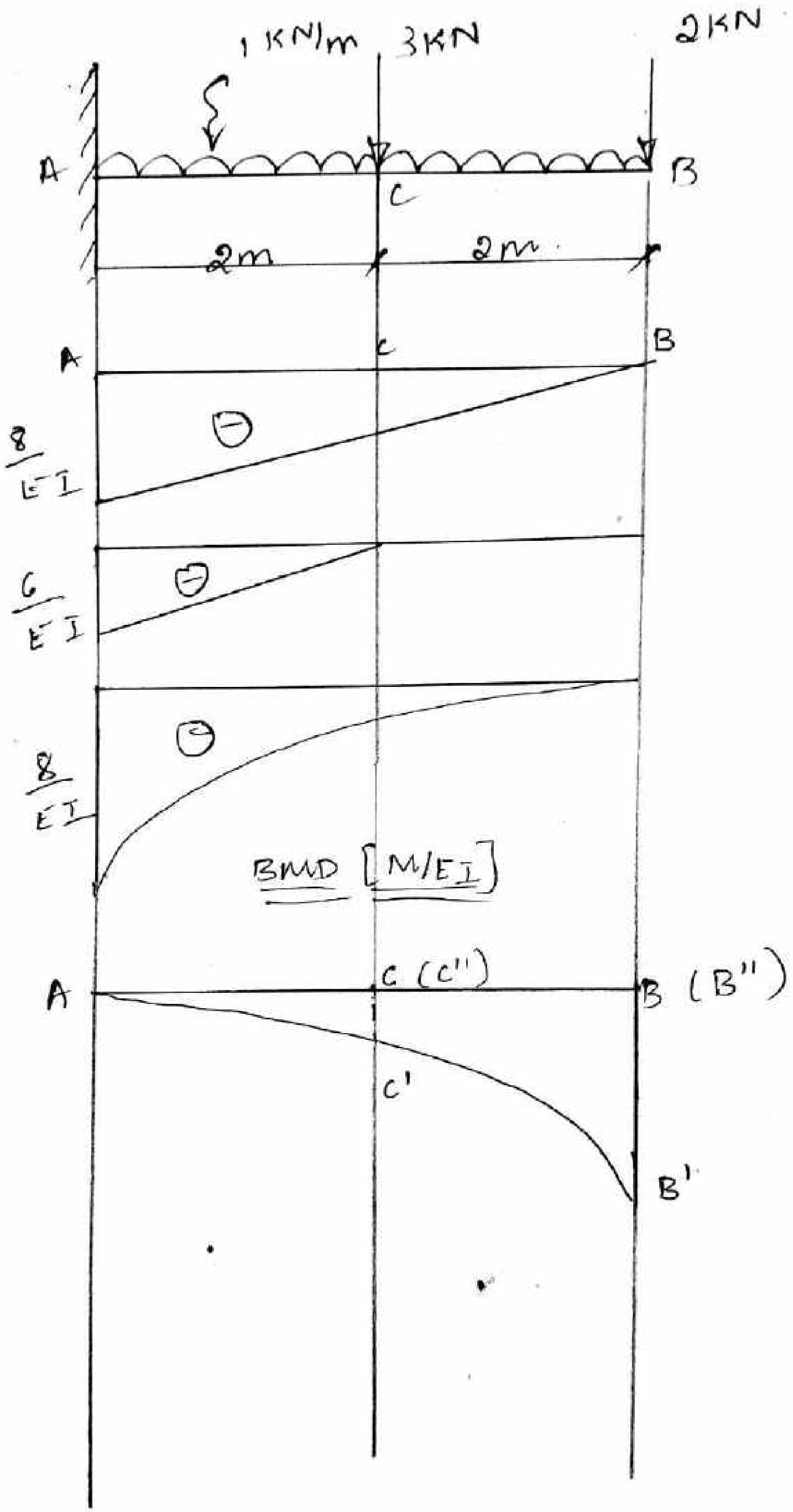
Deflection (BB'):-

From Mohr's 2nd theorem,

$BB' = B'B'' =$ Moment of area of M/EI diagram b/w A and B about B.

$BB' = B'B'' = -\left[\frac{1}{3}\times5\times\frac{25}{EI}\right]\left[\frac{3}{4}\times5\right]$

$$\boxed{BB' = \frac{-156.25}{EI}}$$


1 KN/m
3KN
2KN
A
B
C
2m
2m
A
C
B
8/EI
6/EI
8/EI
BMD [M/EI]
A
C (C'')
B (B'')
C'
B'

11). Find the slope and deflection at the free end as shown in fig:-

BMD:-

BM_A due to 2KN = $(2\times4) = 8$ KNm.

BM_A due to 3KN = $(3\times2) = 6$ KNm

BM_A due to UDL = $(1\times4)(4/2) = 8$ KNm.

Slope:-

From Mohr's I^{st} theorem,

$\theta_A - \theta_B$ = Area of $M/_{EI}$ diagram b/w A & B.

$$0-\theta_B = -\left[\frac{1}{2}\times4\times\frac{8}{EI}\right] - \left[\frac{1}{2}\times2\times\frac{6}{EI}\right] - \left[\frac{1}{3}\times4\times\frac{8}{EI}\right]$$

$$-\theta_B = -\frac{32.67}{EI}.$$

$$\therefore \boxed{\theta_B = \frac{32.67}{EI}}$$

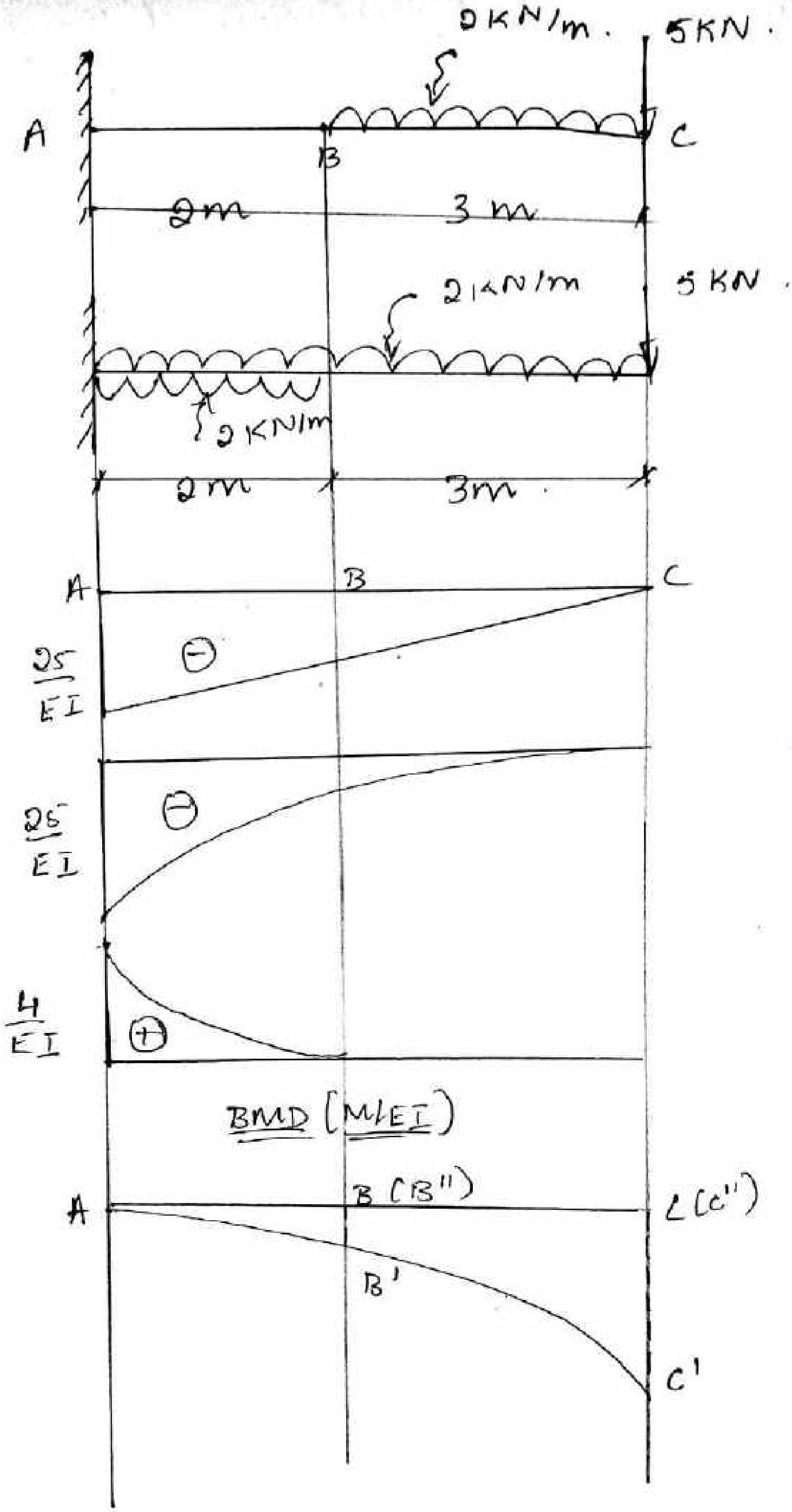
Deflection:-

From Mohr's 2^{nd} theorem,

$B'B' = B'B''$ = Moment of area of $M/_{EI}$ diagram b/w A and B about B.

$$BB' = B'B'' = -\left[\frac{1}{2}\times4\times\frac{8}{EI}\right]\left[\frac{2}{3}\times4\right] - \left[\frac{1}{2}\times2\times\frac{6}{EI}\right]\left[2+\frac{2}{3}\times2\right]$$
$$-\left[\frac{1}{3}\times4\times\frac{8}{EI}\right]\left[3/_4\times4\right]$$

$$\boxed{BB' = \frac{-94.67}{EI}}$$



2KN/m.
5KN.
A
B
C
2m
3 m
2KN/m
5KN.
2KN/m
2m
3m.
A
B
C
25/EI
25/EI
4/EI
BMD (M/EI)
A
B (B'')
C (C'')
B'
C'

13). Find slope and deflection at free end for the beam shown in fig:-

BMD:-

BM_A due to 5 KN = $(5 \times 5) = 25$ KNm

BM_A due to downward UDL = $(2 \times 5)(5/2) = 25$ KNm

BM_A due to upward UDL = $(2 \times 2)(2/2) = 4$ KNm.

Slope:-

From Mohr's 1st theorem,

$\theta_A - \theta_C$ = Area of M/EI diagram b/w A and C.

$$0 - \theta_C = -\left[\frac{1}{2} \times 5 \times \frac{25}{EI}\right] - \left[\frac{1}{3} \times 5 \times \frac{25}{EI}\right] + \left[\frac{1}{2} \times 2 \times \frac{4}{EI}\right]$$

$$\boxed{\theta_C = \frac{100.167}{EI}}$$

Deflection:-

From Mohr's 2nd theorem,

$CC' = C'C''$ = Moment of area of M/EI diagram between A and C about C.

$$CC' = C'C'' = -\left[\frac{1}{2} \times 5 \times \frac{25}{EI}\right]\left[\frac{2}{3} \times 5\right] - \left[\frac{1}{3} \times 5 \times \frac{25}{EI}\right]\left[\frac{3}{4} \times 5\right] + \left[\frac{1}{3} \times 2 \times \frac{4}{EI}\right]\left[3 + \frac{3}{4} \times 2\right]$$

$$\boxed{CC' = \frac{-358.58}{EI}}$$

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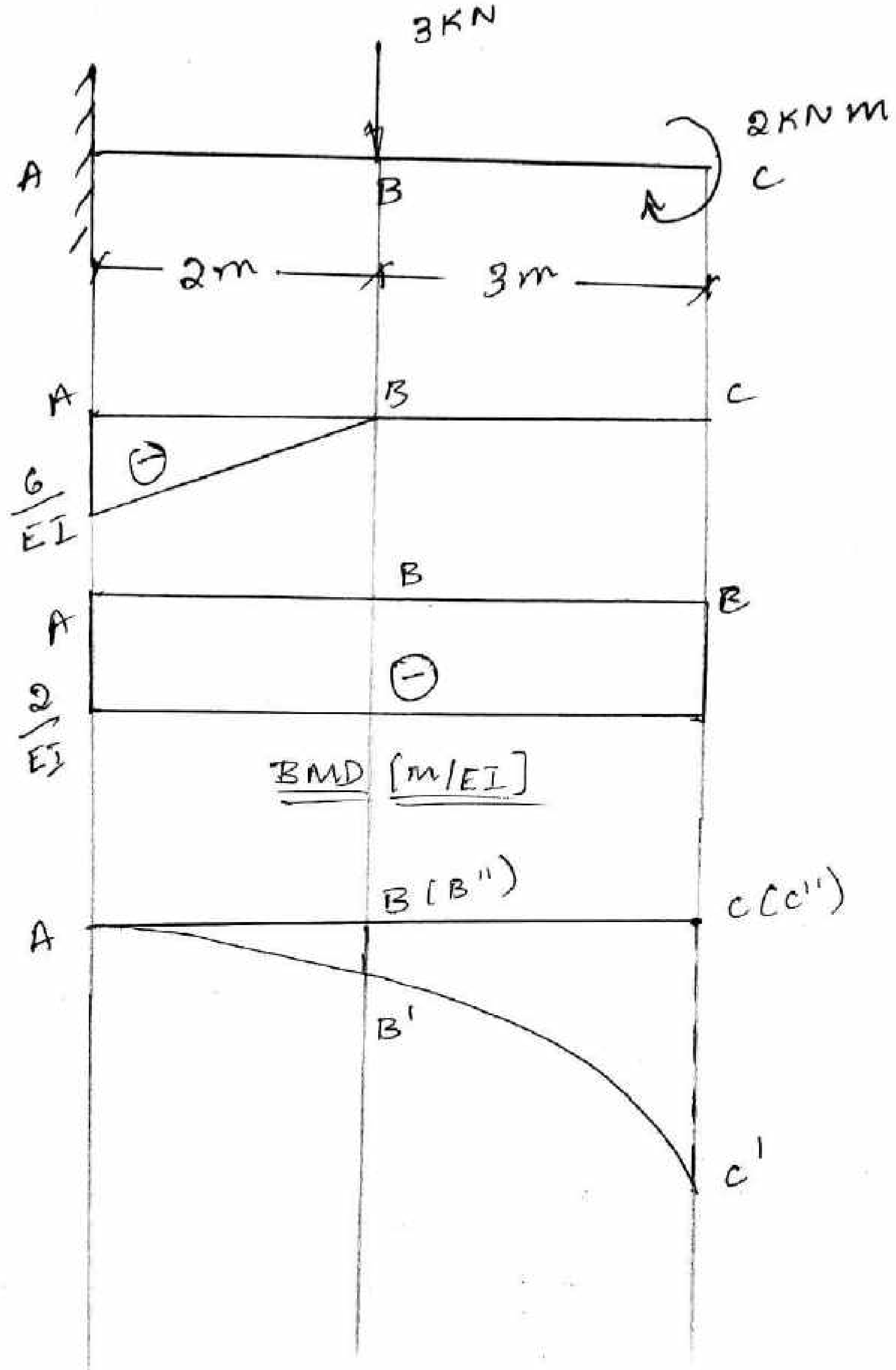

3KN
2KNm
A
B
C
2m
3m
A
B
C
6/EI
B
A
B
2/EI
BMD [M/EI]
A
B (B'')
C (C'')
B'
C'

14). Find the Slope and deflection at free end for the beam shown in fig.:-

BMD :-

BM_A due to 3 KN $= (3 \times 2) = 6$ KNm

BM_A due to couple $= 2$ KNm

Slope :-

From Mohr's 1st theorem,

$\theta_A - \theta_c =$ Area of M/EI diagram b/w A & C.

$$\theta_A - \theta_C = -\left[\frac{1}{2} \times 2 \times \frac{6}{EI}\right] - \left[\frac{2}{EI} \times 5\right]$$

$$\boxed{\theta_C = \frac{16}{EI}}$$

Deflection :- (CC')

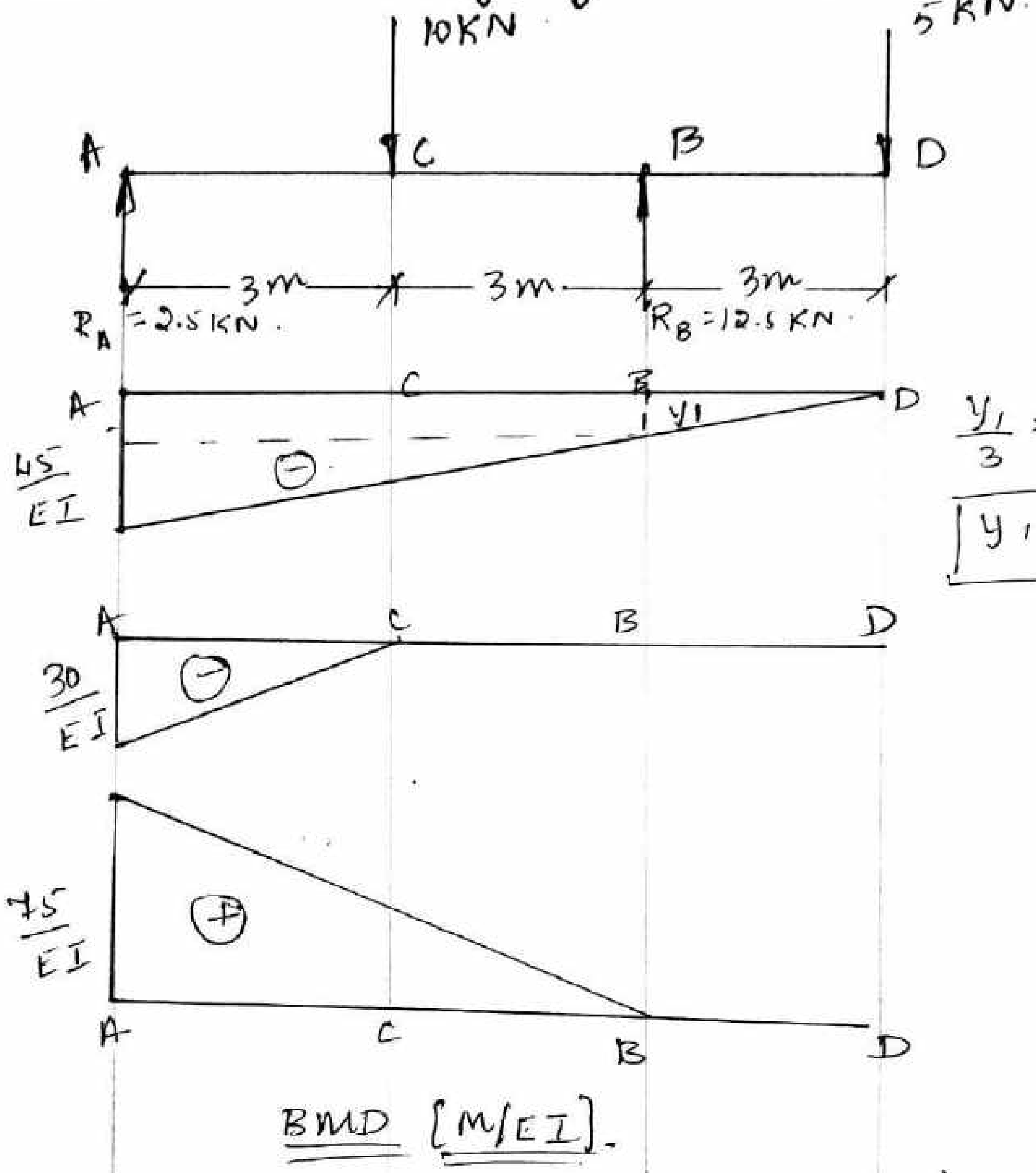
From Mohr's 2nd theorem,

$CC' = c'c'' =$ Moment of area of M/EI diagram b/w A & C about C.

$$CC' = c'c'' = -\left[\frac{1}{2} \times 2 \times \frac{6}{EI}\right]\left[3 + \frac{2}{3} \times 2\right] - \left[\frac{2}{EI} \times 5\right]\left[\frac{5}{2}\right]$$

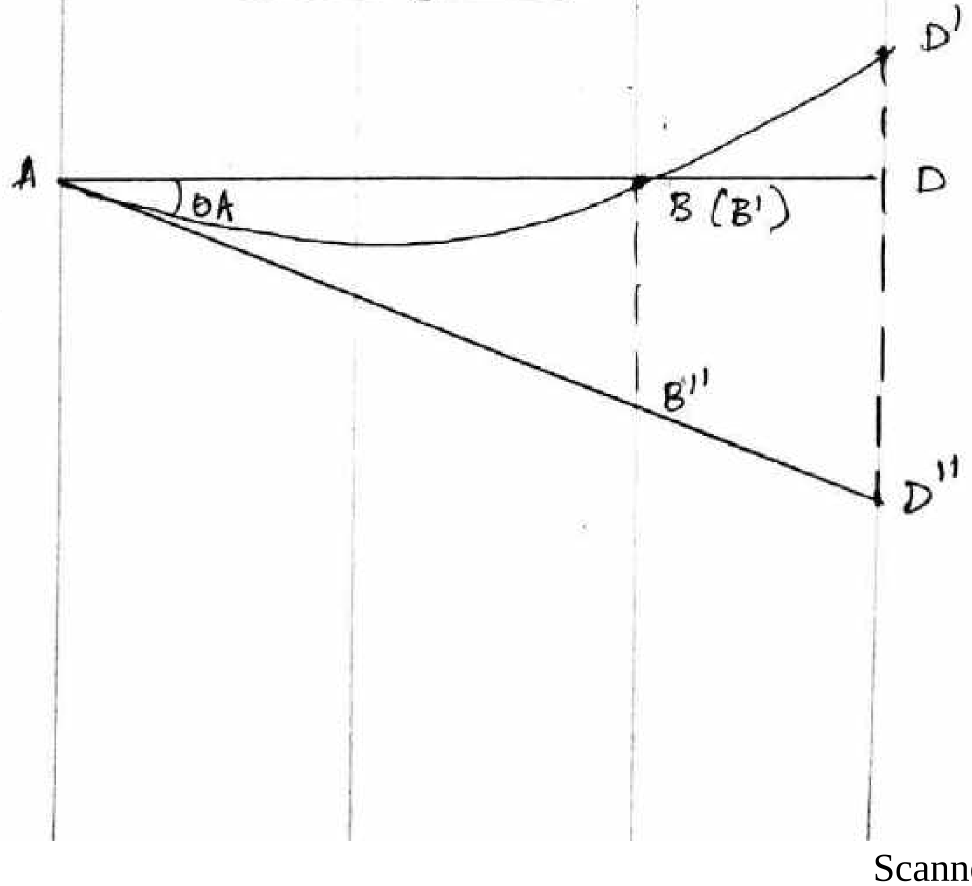
$$\boxed{CC' = \frac{-51}{EI}}$$

Problems on Overhanging beam:-



$$\frac{y_1}{3} = \frac{45}{9EI}$$

$$\boxed{y_1 = \frac{15}{EI}}$$



Q15). Using Moment-Area method, determine slope at A and B and deflection at D :-

Support reactions :-

$\Sigma M_A = 0$

$(10\times3) - (R_B\times6) + 5\times9 = 0$

$\boxed{R_B = 12.5\,KN}$

$\Sigma F_y = 0$

$R_A - 10 + R_B - 5 = 0$

$\boxed{R_A = 2.5\,KN}$

BMD :-

BM_A due to 5KN = $5\times9 = 45$ KNm

BM_A due to 10KN = $10\times3 = 30$ KNm

BM_A due to $R_B = (12.5\times6) = 75$ KNm

Slope :-

Slope at A :-

From Mohr's 2nd theorem,

$B'B''$ = Moment of area of M/EI diagram b/w A & B about B.

$B'B'' = -\left[\frac{15}{EI}\times6\right]\left(6/2\right) - \left(\frac{1}{2}\times6\times\frac{30}{EI}\right)\left(2/3\times6\right) - \left(\frac{1}{2}\times3\times\frac{30}{EI}\right)\left(3+2/3\times3\right) + \left(1/2\times6\times75/EI\right)\left(2/3\times6\right)$

$\boxed{B'B'' = 45/EI}$

$\theta_A = \frac{B'B''}{AB}$

$= \frac{45}{6EI}$

$\boxed{\theta_A = \frac{7.5}{EI}}$

Slope at B :-

From Mohr's 1st theorem,

$\theta_A - \theta_B$ = Area of M/EI diagram b/w A & B.

$\frac{7.5}{EI} - \theta_B = -\left[\frac{1}{2}\times6\times\frac{30}{EI}\right] - \left(\frac{15}{EI}\times6\right) - \left[\frac{1}{2}\times3\times\frac{30}{EI}\right] + \left(\frac{1}{2}\times6\times\frac{75}{EI}\right)$

$\frac{7.5}{EI} - \theta_B = 0$

$\boxed{\theta_B = \frac{7.5}{EI}}$

Deflection :- (DD')

From Mohr's 2nd theorem,

$D'D''$ = Moment of area of M/EI diagram b/w A & D about D

$D'D'' = -\left(\frac{1}{2}\times9\times\frac{45}{EI}\right)\left(2/3\times9\right) - \left(\frac{1}{2}\times3\times\frac{30}{EI}\right)\left(6+2/3\times3\right) + \left(\frac{1}{2}\times6\times\frac{75}{EI}\right)\left(2/3\times6+3\right)$

$\boxed{D'D'' = 0}$

$\theta_A = \frac{DD''}{AD}$

$DD'' = \theta_A\,(AD)$

$= \frac{7.5}{EI}\,(9)$

$\boxed{DD'' = \frac{67.5}{EI}}$

$DD' = D'D'' - DD''$

$= 0 - 67.5/EI$

$\boxed{DD' = -67.5/EI}$

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MODULE-04

ENERGY PRINCIPLES AND ENERGY THEOREMS

Energy Principles And Energy theorems Principal of Virtual displacements, Principle of virtual forces, strain energy & complimentary energy, strain energy due to direct force, Strain energy due to bending, Deflection of determinate beams & trusses using total strain energy, Deflection at the point of application of single load, Castiglianos theorems and its application of to estimate the deflections of trusses, bent frames, Special applications - Dummy unit load methods.

Strain Energy:-

"Energy stored in an elastic body under loading or due to deformation is called as strain energy"

Strain energy = Work done by External loads.

Law of Conservation of Energy:

Energy in a system may take various forms like kinetic, Potential, heat, light etc

The law of conservation of energy states that "Energy can neither be created nor be destroyed, however energy can be converted from one form to another form of energy"

$\therefore$ Sum of all the energies in the system is constant.

Eg: A mass hanging from the ceiling will have a kinetic energy equal to zero. If the cord breaks, the mass will rapidly increase its kinetic energy. This kinetic energy was somehow stored in the mass whent it was hanging. The energy was hidden but has the potential to reappear as kinetic energy. The stored energy is called potential energy.

Conservation of energy tells us that the total energy of the system is conserved, and in this case the sum of kinetic & potential energy must be constant. This means that every change in the kinetic energy of a system must be accomponied by an equal but apposite change in the potential energy.

Theorem of Minimum Potential Energy.

The total potential energy of an elastic body is defined as the sum of strain energy & work potential

i.e $\boxed{\pi = U + W_p}$

where $U \rightarrow$ Strain energy

$W_p \rightarrow$ Work potential of applied loads

The above equation means that for equilibrium to be ensured, the total potential energy must be stationary. For elastic systems. The total potential energy is not only stationary but also absolute minimum.

We can relate the variations of the workdone by the loads "W" and of the work potential "W_p"

$$\boxed{\delta W = -\delta W_p}$$

The principle of minimum potential energy is

$$\delta\pi = \delta U + \delta W_p$$
$$= \delta U - \delta W = 0$$

Statement : " Of all possible displacement configurations a body can assume which satisfy compatability & the constraints or kinematic boundary conditions, the configuration satisfying equilibrium makes the potential energy assume a minimum value"

It indicates that A structure or body shall deform or displace to a position that minimizes the total potential energy with the lost potential energy being converted into kinetic energy.

Principle of Virtual Work :

The principal of virtual work is the most fundamental and powerful tool available for the analysis of statically indeterminate structures and has the advantage of being able to deal with conditions other than those in the elastic range.

The principle of virtual work states that " The stress, body force [gravity, weight] & traction [Area force, wind pressure, fluid pressure] are in equilibrium if and only if the internal virtual work [W_i] equal to the external virtual work [W_e] for every virtual displacement field"

i.e $\delta W = \delta W_e + \delta W_i = 0$

i.e $\delta W_e = \delta W_i$

Virtual work is defined as work done by real forces acting through virtual displacements. These virtual displacements need not be real and can be virtual [imaginary]

Strain Energy & Complimentary Energy

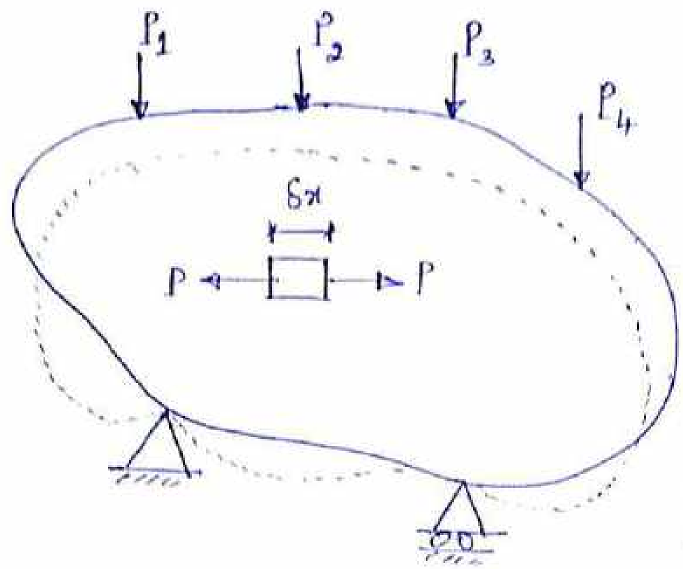


Fig: 1) General Structural System.

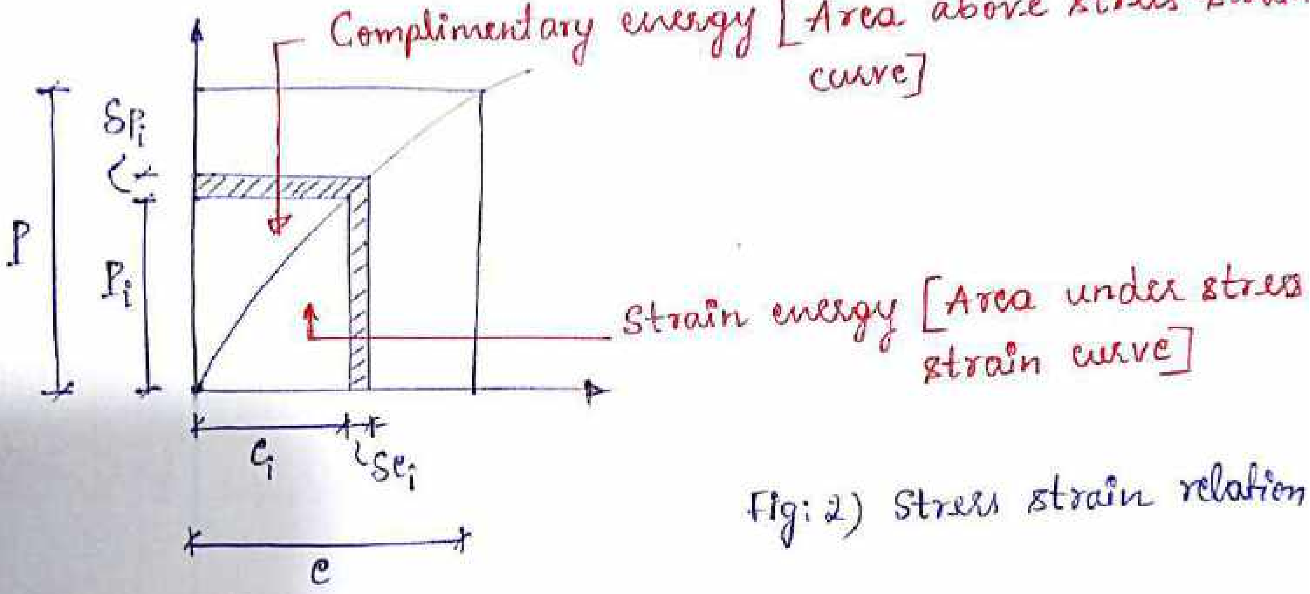


Fig: 2) Stress strain relation

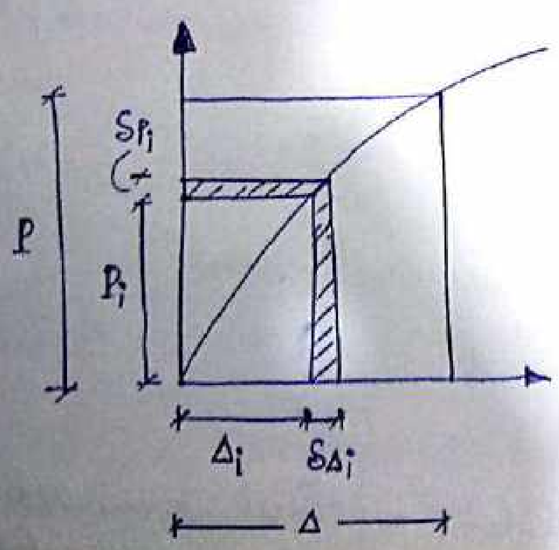


Fig: 3) Load v/s deformation curve

Consider general structural system as shown in fig: 01

Let c/s^{nal} area of the element be "δa" & its length "dx"

The stress in the element gradually increases from zero to final value 'P' as strain increases from zero to its final value 'e'

Let stress strain relation be shown in fig (2)

Let stress be P_i, hence workdone on the element when strain δe_i takes place.

$$= \text{Force} \times \text{displacement}$$

$$= P_i . \delta a \times \delta e_i . \delta x$$

Since, $\delta a . \delta x = \delta V$, where '$V$' is the volume of the element.

$$= P_i \, \delta e_i . \delta v$$

$\therefore$ Strain energy stored in the element $= \int_0^e P_i \, \delta e_i . \delta v$

$=$ Area under stress-strain curve $\times \delta v$

If the stress-strain curve is linear, $SE = \frac{1}{2} \times P \times e \times \delta v$

$\therefore$ Strain energy stored in the structure $U = \int \frac{1}{2} P e \, dv$

$$\boxed{U = \int \frac{1}{2} \text{ Stress} \times \text{Strain} \times dv} \longrightarrow (1)$$

Fig (3) shows Load v/s Deformation relation.

Let, during deformation the load acting be 'P_i' & deformation be '$\delta\Delta_i$'

Then, Workdone $= \int P_i \, \delta\Delta_i$

= Area under the load deformation curve

$= \frac{1}{2} \times P \times \Delta$ (in case of linear elasticity problem)

For "n" no of loads.

$$= \int_0^n \frac{1}{2} P\Delta = \Sigma \frac{1}{2} P\Delta$$

Complimentary energy at any instant during deformation of the element is given by $e_i \cdot \delta P_i \cdot dv$

Hence the complimentary energy of the element when final deformation takes place

$$C \cdot E = \int_0^{P_i} e_i \, \delta P_i \cdot \delta v$$

= Area above stress-strain curve $\times dv$

$= \int \frac{1}{2} \times P \times e \times dv$ in case of linear elasticity problems

Thus complimentary energy of the entire structure

$$\boxed{V_c = \int \frac{1}{2} P e \, dv} \longrightarrow (2)$$ in case of linear elasticity problem.

From eqn (1) & (2) we can conclude that in case of linear elasticity problem, Strain energy & complimentary energy are equal.

III^{ly} Complimentary workdone is given by

$$= \int_0^{P_i} \Delta_i \, \delta P_i$$

In case of linear elasticity problems.

$$\text{Workdone} = \frac{1}{2} \times \Delta \times P$$

If there are 'n' no of loads.

$$\boxed{\text{Workdone} = \Sigma \frac{1}{2} \Delta P}$$

Derive an expression for strain energy due to Axial load, bending.

Soln

Strain Energy due to axial force:

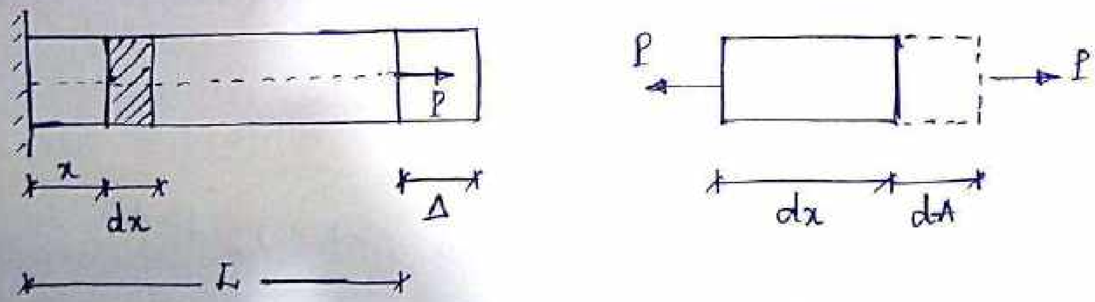


Fig: Member subjected to axial force 'P'

Consider a member of constant cross sectional area 'A' subjected to axial force 'P' through the centroid of the cross section as shown in above fig.

W.K.T Stress = $\boxed{\sigma = P/A}$ ⟶ (1)

(7)

Under the action of axial load 'P' applied at one end gradually, the beam get elongated by 'Δ'
The incremental elongation '$d\Delta$' of small element of length of beam 'dx' is given by

$$\epsilon = \frac{d\Delta}{dx}$$

$$d\Delta = \epsilon \, dx.$$

$$\left\{ \text{W.K.T} \quad E = \frac{\sigma}{\epsilon} \Rightarrow \epsilon = \frac{\sigma}{E} \right\}$$

$$\boxed{d\Delta = \frac{\sigma}{E} \cdot dx} \longrightarrow (2)$$

Substituting eqn (1) in (2)

$$d\Delta = \left(\frac{P}{AE}\right) dx.$$

The total elongation of the member of length 'L' may be obtained by integration.

$$\boxed{\Delta = \int_0^L \frac{P}{AE} \cdot dx} \longrightarrow (3)$$

Workdone by external loads, $\boxed{W = \frac{1}{2} \cdot P \cdot \Delta}$

S.E = External Work done.

$$\boxed{U = W = \frac{1}{2} \cdot P \cdot \Delta} \longrightarrow (4)$$

Substituting eqn (3) in (4)

$$U = \frac{1}{2} P \left[\int_0^L \frac{P}{AE} dx \right] = \int_0^L \frac{P^2}{2AE} \cdot dx$$

$$= \frac{P^2}{2AE}\left[x\right]_0^L = \frac{P^2L}{2AE}$$

$$\boxed{U = \frac{P^2L}{2AE}} \longrightarrow$$ for a straight prismatic member of length 'L' subjected to axial load 'P'

Strain energy due to Bending Moment:

Consider a small segment of prismatic beam of length 'dx' subjected to bending moment 'M' as shown in fig. Now one cross section rotates about another cross section by a small amount $d\theta$.

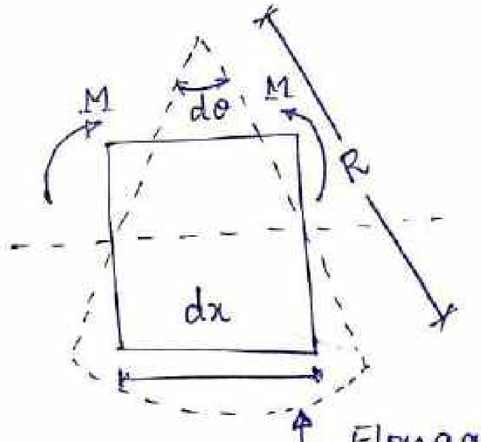


Fig: Member subjected to BM

From fig. $\boxed{dx = R \cdot d\theta} \longrightarrow (1)$

From bending eqn.

$$\frac{M}{I} = \frac{E}{R}$$

$$\boxed{R = \frac{EI}{M}} \longrightarrow (2)$$

$$\frac{1}{R} = \frac{M}{EI}$$

Substituting eqn (2) in (1)

$$dx = \frac{EI}{M} \cdot d\theta$$

$$\boxed{d\theta = \frac{M}{EI} \cdot dx} \longrightarrow (3)$$

When 'R' is the radius of curvature of the bent beam and EI is the flexural rigidity of the beam.

Workdone by the moment 'M' while rotating through angle 'dθ' will be stored in the segment of beam as strain energy. Hence.

$$\boxed{du = \tfrac{1}{2} \cdot M \cdot d\theta} \longrightarrow (4)$$

Substituting eqn (3) in (4)

$$du = \tfrac{1}{2} \cdot \tfrac{M^2}{EI} \cdot dx$$

The energy stored in the complete beam of span 'L'

$$\boxed{U = \int_0^L \frac{M^2}{2EI} \cdot dx}$$

Deflection of Beams & Trusses by Strain energy.

Procedure for finding deflection by strain energy method.

1. This method is used only when a single point load is acting on the beam or frame.
2. Find support reactions and compute moment at section x-x.
3. Apply strain energy method for entire beam.

$$U = \int_0^L \frac{M^2}{2EI}\, dx .$$

4. Apply law of conservation of energy.

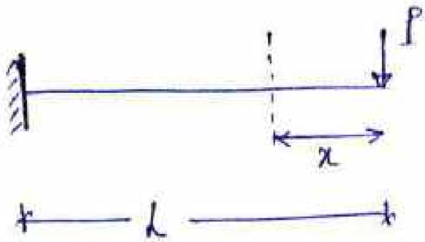
$$U = \frac{1}{2} P \Delta$$

5. Find deflection 'Δ'

Limitations of strain energy method:

1. This method is used to find deflection below the point load only
2. This method is applicable when a single concentrated load is acting on the beam.

Eg 1) Using Strain Energy method determine the deflection of the free end of a cantilever of length 'L' subjected to a concentrated load P at the free end.



Soln:

B.M at a distance 'x' from free end is

$$M = P.x$$

$$\therefore \; SE = \int_0^L \frac{M^2}{2EI} \cdot dx$$

$$= \int_0^L \frac{P^2 x^2}{2EI} \cdot dx$$

$$= \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^L$$

$$= \frac{P^2}{2EI} \left[\frac{L^3}{3} \right]$$

$$SE = \frac{P^2 L^3}{6EI}$$

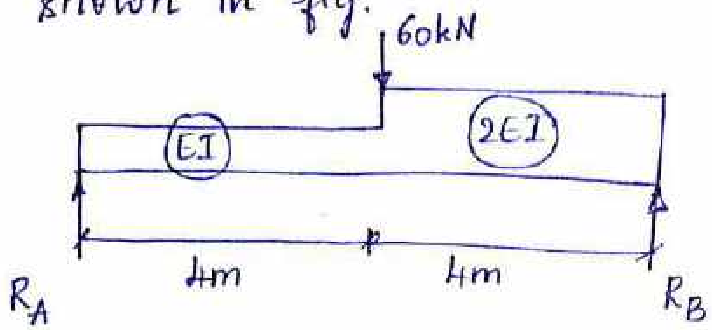
Work done by load $= \frac{1}{2} \cdot P\Delta$

SE = Work done by external loads.

$$\frac{P^2 L^3}{6EI} = \frac{1}{2} \cdot P\Delta$$

$$\therefore \boxed{\Delta = \frac{PL^3}{3EI}}$$

Eg. 2) Determine the deflection under 60 kN load in the beam as shown in fig.



Soln: $R_A = R_B = 30\text{ kN}$.

$\therefore$ B.M at any distance 'x' from A [or B] = $\underline{30x \text{ kN}}$

$$S.E = \int_0^4 \frac{(30x)^2}{2EI}\cdot dx + \int_0^4 \frac{(30x)^2}{2\times 2EI}\cdot dx$$

$$= \frac{900}{2EI}\left[\frac{x^3}{3}\right]_0^4 + \frac{900}{4EI}\left[\frac{x^3}{3}\right]_0^4$$

$$= \left[\frac{9600}{EI} + \frac{4800}{EI}\right]$$

$$S.E = \frac{14,400}{EI}$$

$$W = \tfrac{1}{2}\cdot P\Delta \quad = \tfrac{1}{2}\times 60\times\Delta$$

$$SE = W$$

$$\frac{14,400}{EI} = \tfrac{1}{2}\times 60\times\Delta$$

$$\boxed{\Delta = \frac{480}{EI}}$$

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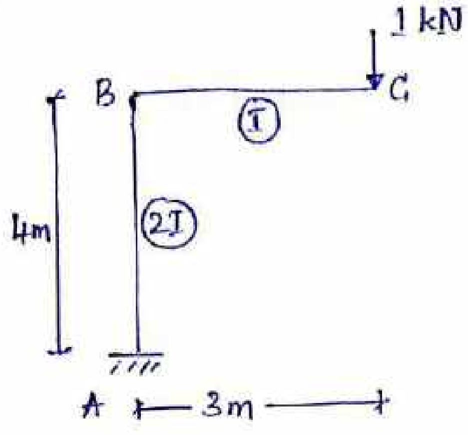
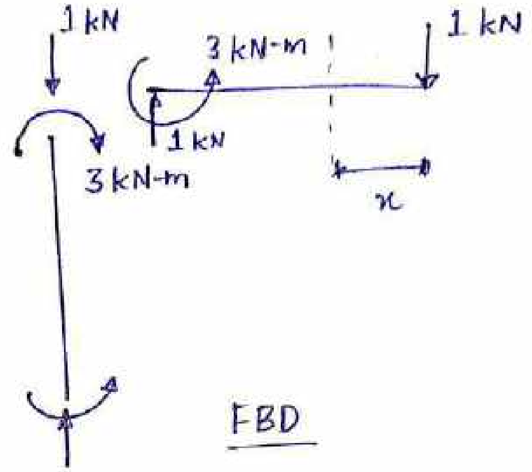
Eg 3) Determine the vertical deflection of point C in the frame shown in fig. Given $E = 200\ kN/mm^2$

$I = 30 \times 10^6\ mm^4$





FBD

Soln

Portion	Origin	Limit	Expression
BC	C	0–3	$1x = x$
BA	B	0–4	3

$$S.E = \int_0^3 \frac{x^2}{2EI}\,dx + \int_0^4 \frac{3^2}{2EI}\cdot dx = \frac{1}{2EI}\left[\frac{x^3}{3}\right]_0^3 + \frac{9}{4EI}\left[x\right]_0^4$$

$$= \frac{1}{6EI}\left[3^3 - 0\right] + \frac{9}{4EI}\left[4-0\right]$$

$$S.E = \frac{13.5}{EI}$$

$$W = \tfrac{1}{2} \times P \times \Delta = \tfrac{1}{2} \times 1 \times \Delta$$

$$\therefore \boxed{W = \Delta/2}$$

$$S.E = W$$

$$\frac{13.5}{EI} = \frac{\Delta}{2}$$

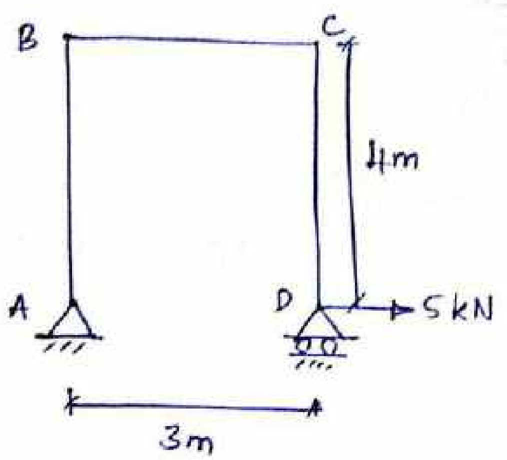
$$\Delta = \frac{27}{6000} = 0.045m \Rightarrow 4.5mm$$

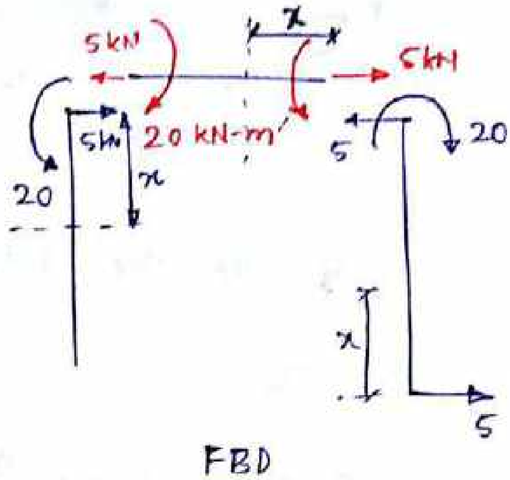
$$EI = 200 \times 30 \times 10^6\ kN\text{-}mm^2$$

$$EI = 200 \times 30 \times 10^6 \times 10^{-6}\ kN\text{-}m^2$$

$$EI = 6000\ kN\text{-}m^2$$

Eg 4) Determine the horizontal displacement of the roller end D of the portal frame shown in fig.
Take $EI = 8000\ kN\text{-}m^2$ throughout.





FBD

Portion	CD	BC	AB
Origin	D	C	B
Limit	0-4	0-3	0-4
M_x	$5x$	20	$5(4-x)$

$$\therefore S.E = \int_0^4 \frac{(5x)^2}{2EI}dx + \int_0^3 \frac{20^2}{2EI}dx + \int_0^4 \frac{(20-5x)^2}{2EI}dx$$

$$= \frac{1}{2EI}\left[\frac{25x^3}{3}\right]_0^4 + \frac{1}{2EI}\left[400x\right]_0^3 + \frac{1}{2EI}\left[400x - \frac{200x^2}{2} + \frac{25x^3}{3}\right]_0^4$$

$$S.E = \frac{266.67}{EI} + \frac{600}{EI} + \frac{1}{2EI}\left[1600 - 1600 + \frac{25 \times 64}{3}\right] = \frac{1133.33}{EI}$$

$$W = \frac{1}{2} \times P \times \Delta = \frac{1}{2} \times 5 \times \Delta = 2.5\Delta$$

$$S.E = W$$

$$\frac{1133.33}{EI} = 2.5\Delta \qquad \Delta = \frac{453.33}{EI} = \frac{453.33}{8000} = 0.0567\,m$$

$$\therefore \boxed{\Delta = 56.7\ mm}$$

* CASTIGLIANO'S THEOREMS :-

First Theorem :- [deflection based on strain energy of the structure]

"In a linearly elastic structure, partial derivative of the strain energy with respect to a load is equal to the deflection of the point where the load is acting, the deflection being measured in the direction of the load"

Mathematically,

$$\frac{dU}{dP_i} = \Delta_i \quad , \quad \frac{dU}{dM_j} = \theta_j$$

where, U = Total strain energy

P_i, M_j = loads

Δ_i, θ_j = deflections

Second Theorem :- [deflection based on complimentary strain energy of the structure].

"In an elastic system, the partial derivative of the complimentary strain energy $[U^*]$ of the structure with respect to any particular force equals the deflection of the point of application of that force in the direction of its line of action"

Mathematically, $\frac{\delta U^*}{\delta P_n} = u_n$.

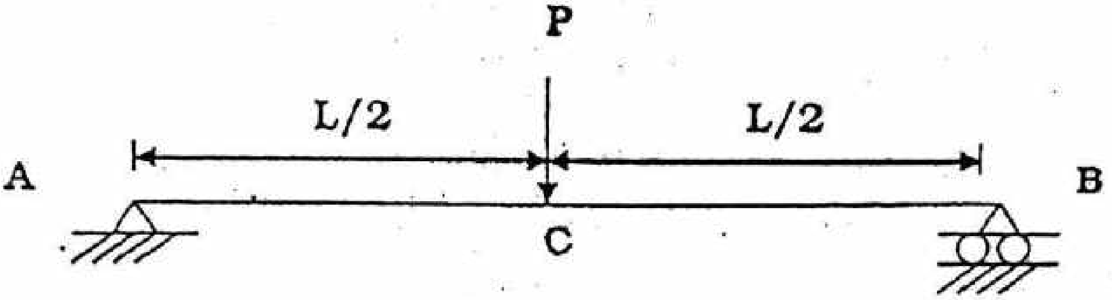
3.5 Procedure for finding deflection using Castigliano's theorem:

1. Find reactions and compute moment M about X-X
2. Calculate $\frac{\partial M}{\partial P}$ where P is a Point load
3. If Point load is not given in the problem, then assume a dummy point load P at the point where deflection is required.
4. Apply Castigliano's theorem, $\Delta = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}dx$

5. In case of dummy point load, put $P = 0$ in the above equation.
6. To find slope at a point, apply a dummy moment P at that point and apply Castigliano's theorem.

$\Delta = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}dx$ and then put $P = 0$ for dummy moment.

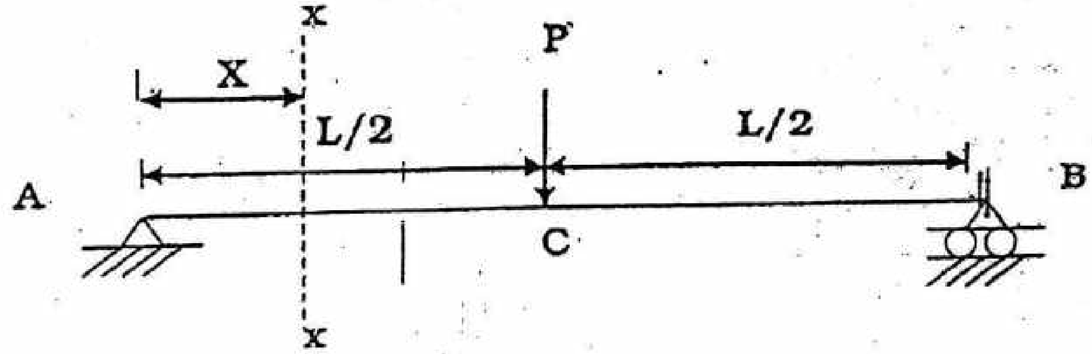
Problem 1: Determine the deflection at the load point for the beam shown in figure by using Castigliano's theorem.



Solution:

Let X be the distance from A.

Due to symmetry, consider only half portion i.e. AC portion. i.e. $X = 0$ to $\frac{L}{2}$.



Reactions:

$$R_A = R_B = \frac{P}{2}$$

Calculating Moment at x-x:

$$M_x = R_A \times X = \frac{PX}{2}$$

Calculating $\frac{\partial M}{\partial P}$:

$$\frac{\partial M}{\partial P} = \frac{\partial(\frac{PX}{2})}{\partial P} = \frac{X}{2}$$

Calculating deflection at C load point:

Apply Castigliano's theorem, for the entire beam i.e $2 \times \Delta_{AC}$.

$$\Delta_C = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}dx = 2 \times \int_0^{L/2} \frac{M}{EI}\frac{\partial M}{\partial P}dx$$

$$= 2 \times \int_0^{L/2} \frac{\frac{PX}{2}}{EI}\frac{X}{2}dx$$

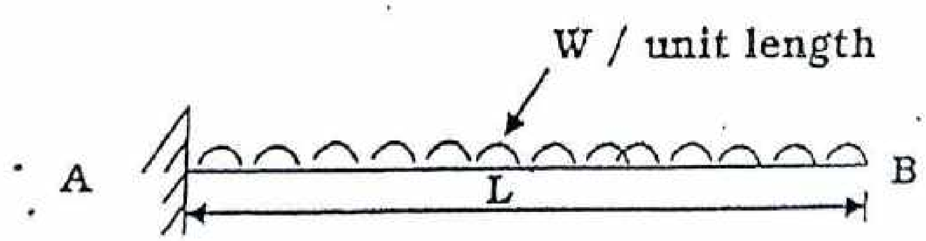
$$= \frac{P}{2EI}\int_0^{L/2}\frac{X^2}{EI}dx$$

$$= \frac{P}{2EI}\left[\frac{X^3}{3}\right]_0^{L/2}$$

$$= \frac{P}{2EI}\left[\frac{(L/2)^3}{3}\right]$$

$$\therefore \Delta_C = \frac{PL^3}{48EI} \text{ acting downwards}$$

Problem 2: Find the vertical deflection and slope at free end for the cantilever beam shown in figure by using Castigliano's theorem.



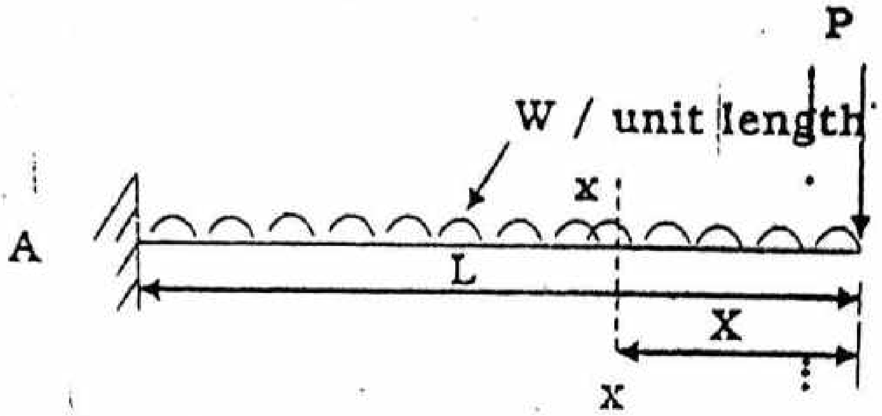
Solution:

Finding vertical deflection:

To find vertical deflection at free end for the beam, we need to apply a dummy point load P at B.

Let X be the distance from B.

Limits for is $X = 0$ to L.



Calculating Moment at x-x:

$$M_x = -P \times X - W \times X \times \frac{X}{2} = PX + \frac{WX^2}{2}$$

$$= -\left(PX + \frac{wx^2}{2}\right)$$

Calculating $\frac{\partial M}{\partial P}$: differentiate wrt P

$$\frac{\partial M}{\partial P} = \frac{\partial(PX + \frac{WX^2}{2})}{\partial P} = -x$$

Calculating deflection at free end B:

Apply Castigliano's theorem,

$$\Delta_B = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}dx = -\int_0^L \left(\frac{PX + \frac{WX^2}{2}}{EI}\right)(-X)dx$$

$$= \frac{1}{EI}\int_0^L (PX^2 + \frac{WX^3}{2})dx$$

Now for dummy load P, put $P = 0$

$$\Delta_B = \frac{1}{EI}\int_0^L (\frac{WX^3}{2})dx$$

$$= \frac{W}{EI}\left[\frac{X^4}{2x4}\right]_0^L$$

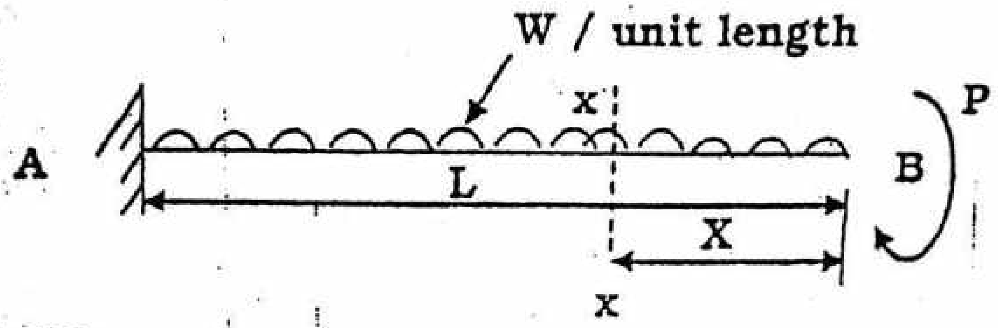
$$\therefore \Delta_B = \frac{WL^4}{8EI} \text{ acting downwards}$$

Finding slope at free end B:

To find slope at free end for the beam, we need to apply a dummy moment P at B.

Let X be the distance from B.

Limits for is $X = 0$ to L



Calculating Moment at x-x:

$$M_x = P + W \times X \times \frac{X}{2} = P + \frac{WX^2}{2} = -\left(P + \frac{wx^2}{2}\right)$$

Calculating $\frac{\partial M}{\partial P}$: differentiate wrt P

$$\frac{\partial M}{\partial P} = \frac{\partial\left(P + \frac{WX^2}{2}\right)}{\partial P} = 1$$

Calculating slope at free end B:

Apply Castigliano's theorem,

$$\theta_B = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}dx = \int_0^L \left(\frac{P + \frac{WX^2}{2}}{EI}(1)\right)dx$$

$$= \frac{1}{EI}\int_0^L \left(P + \frac{WX^2}{2}\right)dx$$

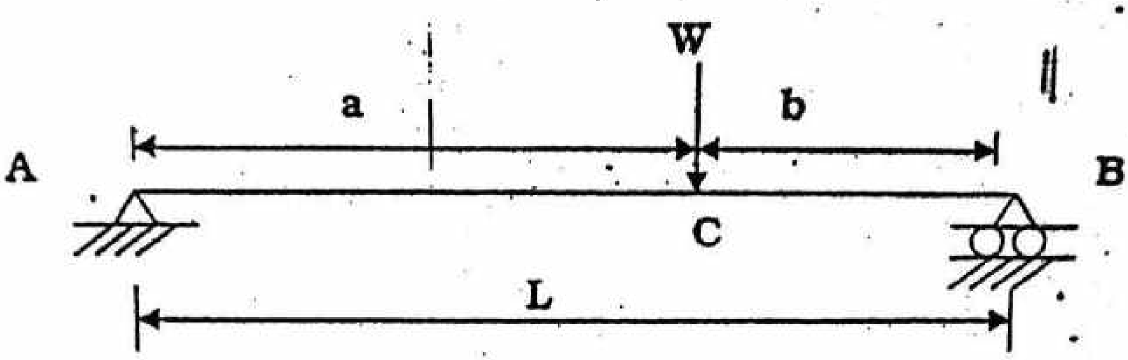
Now for dummy moment P, put $P = 0$

$$\theta_B = \frac{1}{EI}\int_0^L \left(\frac{WX^2}{2}\right)dx$$

$$= \frac{W}{EI}\left[\frac{X^3}{2x3}\right]_0^L$$

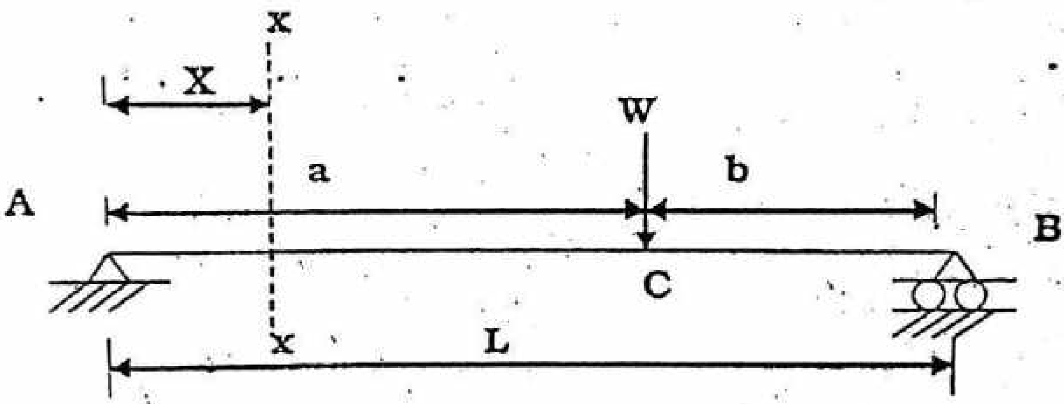
$$\therefore \theta_B = \frac{WL^3}{6EI} \text{ acting clockwise}$$

Problem 3: Using Castigliano's theorem, determine the deflection at the load point for the simply supported beam shown in figure. Take EI = Constant.



Solution:

Let X be the distance from A.



Finding Reactions at supports:

$$\sum M_A = 0$$

$$R_B \times L = Wa$$

$$R_B = \frac{Wa}{L}$$

$$\sum M_A = 0$$

$$R_A + R_B = W$$

$$R_A = W - \frac{Wa}{L} = \frac{Wb}{L}$$

Calculating Moment at x-x:

Portion	AC	CB
Origin	A	B
Limits	0 - a	0 - b
M_x	$\frac{Wb}{L}X$	$\frac{Wa}{L}X$
EI	EI	EI
$\frac{\partial M}{\partial W}$	$\frac{bX}{L}$	$\frac{aX}{L}$

Calculating deflection at C load point:

Apply Castigliano's theorem

$$\Delta_C = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}dx$$

$$= \int_0^a \frac{M}{EI}\frac{\partial M}{\partial W}dx + \int_0^b \frac{M}{EI}\frac{\partial M}{\partial W}dx$$

$$= \int_0^a \left(\frac{\frac{WbX}{L}}{EI}\right)\left(\frac{bX}{L}\right)dx + \int_0^b \left(\frac{\frac{WaX}{L}}{EI}\right)\left(\frac{aX}{L}\right)dx$$

$$= \frac{Wb^2}{EIL^2}\int_0^a X^2 dx + \frac{Wa^2}{EIL^2}\int_0^b X^2 dx$$

$$= \frac{Wb^2}{EIL^2}\left[\frac{X^3}{3}\right]_0^a + \frac{Wa^2}{EIL^2}\left[\frac{X^3}{3}\right]_0^b$$

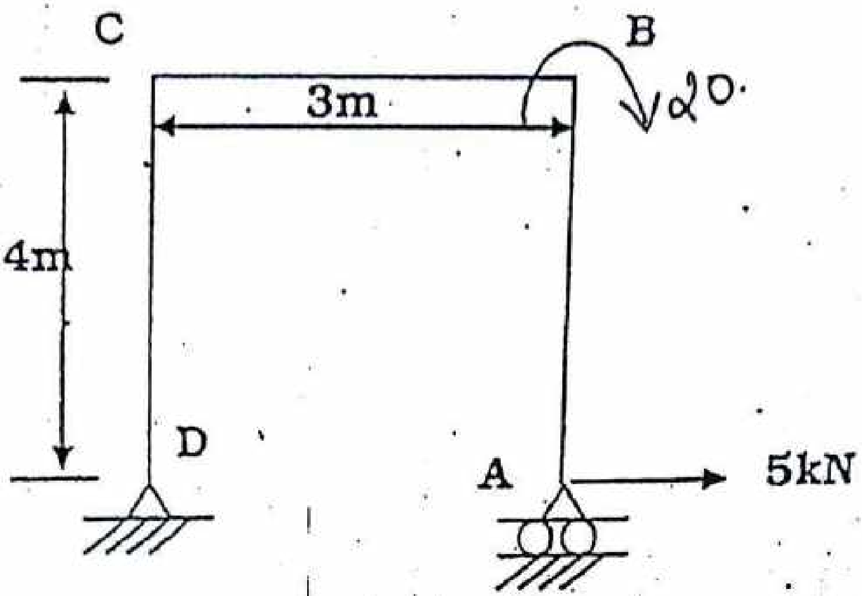
$$= \frac{Wb^2a^3}{3EIL^2} + \frac{Wb^2b^3}{3EIL^2}$$

$$= \frac{Wb^2a^2}{3EIL^2}(a+b)$$

we know that $L = a + b$

$$\therefore \Delta_C = \frac{Wb^2a^2}{3EIL} \text{ acting downwards}$$

Problem 8: Determine the horizontal displacement of the roller support end A of the frame a shown in figure by strain energy method. Take $EI = 8000$ KN-m² (VTU Dec 10, marks 12



Solution:

Finding Horizontal deflection at A:

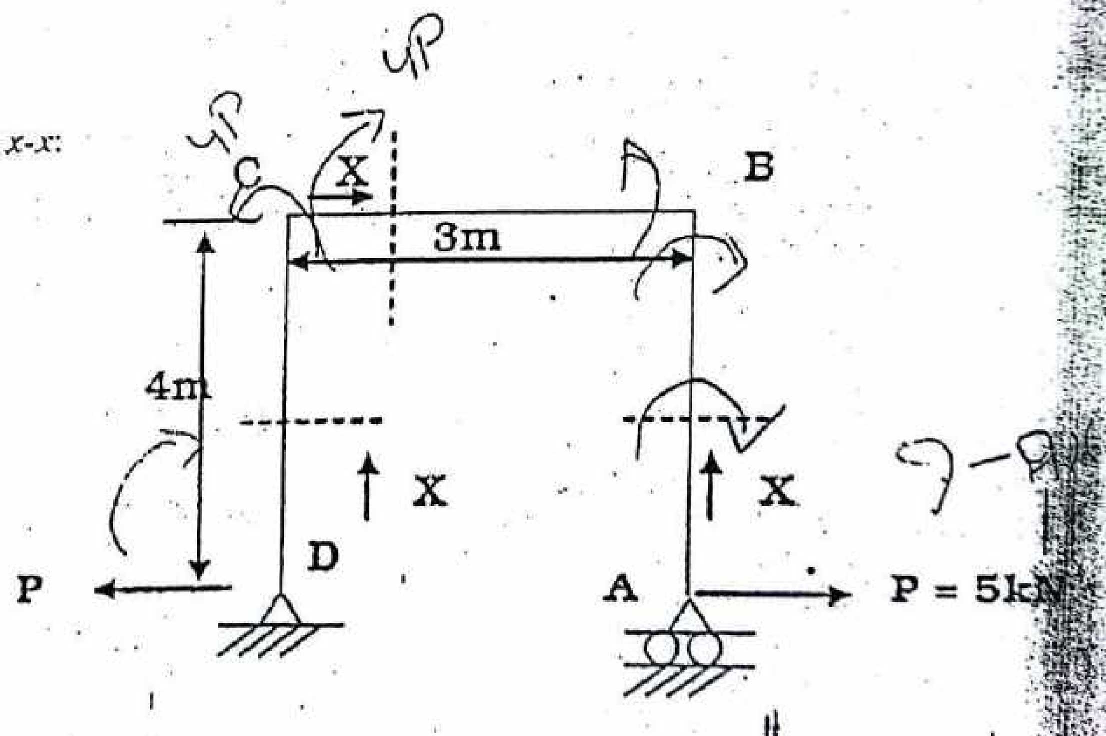
Let $P = 5$kN

Finding reactions:

$\sum H = 0$

$H_D = -5\text{kN} = -P$

Calculating Moment at x-x:



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Portion	AB	BC	CD
Origin	A	C	D
Limits	0 - 4	0-3	0 - 4
M_x	-PX	-4P	-PX
EI	EI	EI	EI
$\frac{\partial M}{\partial P}$	-X	-4	-X

Apply Castigliano's theorem

$$\Delta_A = \int_0^4 \frac{M}{EI}\frac{\partial M}{\partial P}dx + \int_0^3 \frac{M}{EI}\frac{\partial M}{\partial P}dx + \int_0^4 \frac{M}{EI}\frac{\partial M}{\partial P}dx$$

$$= \frac{1}{EI}\int_0^4 (-PX)(-X)dx + \frac{1}{EI}\int_0^3 (-4P)(-4)dx$$

$$+ \frac{1}{EI}\int_0^4 (-PX)(-X)dx$$

$$= \frac{P}{EI}\int_0^4 X^2 dx + \frac{P}{EI}\int_0^3 (16)dx + \frac{P}{EI}\int_0^4 X^2 dx$$

$$= \frac{P}{EI}\left[\frac{X^3}{3}\right]_0^4 + \frac{P}{EI}\left[16X\right]_0^3 + \frac{P}{EI}\left[\frac{X^3}{3}\right]_0^4$$

$$= \frac{P}{EI}\left[\frac{4^3}{3}\right] + \frac{P}{EI}\left[16 \times 3\right] + \frac{P}{EI}\left[\frac{4^3}{3}\right]$$

$$= \frac{21.33P}{EI} + \frac{48P}{EI} + \frac{21.33P}{EI}$$

$$= \frac{90.66P}{EI}$$

Substitute the values of P =5kN and value of EI in the above expression, we get,

$$\Delta_A = \frac{90.66 \times 5}{8000}$$

$$= 0.0566625 \text{ m}$$

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MODULE-05

ARCHES AND CABLE STRUCTURES

CABLE

Cables and arches are closely related to each other and hence they are grouped in this course in the same module. For long span structures (for e.g. in case bridges) engineers commonly use cable or arch construction due to their efficiency. In the first lesson of this module, cables subjected to uniform and concentrated loads are discussed. In the second lesson, arches in general and three hinged arches in particular along with illustrative examples are explained.

Fundamental Characteristic of Cable & Arch

Cables

- ✓ Carry applied loads & develop mostly tensile stresses
- ✓ Loads applied through hangers
- ✓ Cables near the end supporting structures experience bending moments and shear forces

Arches

- ✓ carry applied loads and develop mainly in-plane compressive stresses;
- ✓ Loads applied through ribs
- ✓ Arch sections near the rib supports and
- ✓ Arches, other than three-hinged arches, experience bending moments and shear forces

Example

Cable type structures - Suspension roof, suspension bridges, cable cars, guy-lines, transmission lines, etc.

Arch type structures - Arches, domes, shells, vaults

More information related to the topic

Structure may be classified into rigid and deformable structures depending on change in geometry of the structure while supporting the load. Rigid structures support externally applied loads without appreciable change in their shape (geometry). Beams trusses and frames are examples of rigid structures. Unlike rigid structures, deformable structures undergo changes in their shape according to externally applied loads. However, it should be noted that deformations are still small. Cables and fabric structures are deformable structures. Cables are mainly used to support suspension roofs, bridges and cable car system. They are also used in electrical transmission lines and for structures supporting radio antennas. In the following sections, cables subjected to concentrated load and cables subjected to uniform loads are considered.

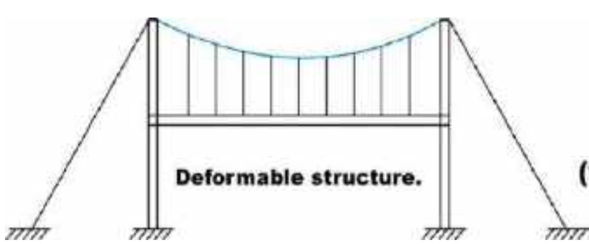


Fig-1

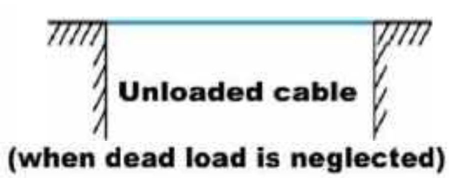


Fig-2

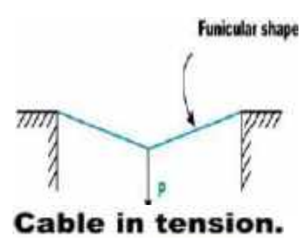
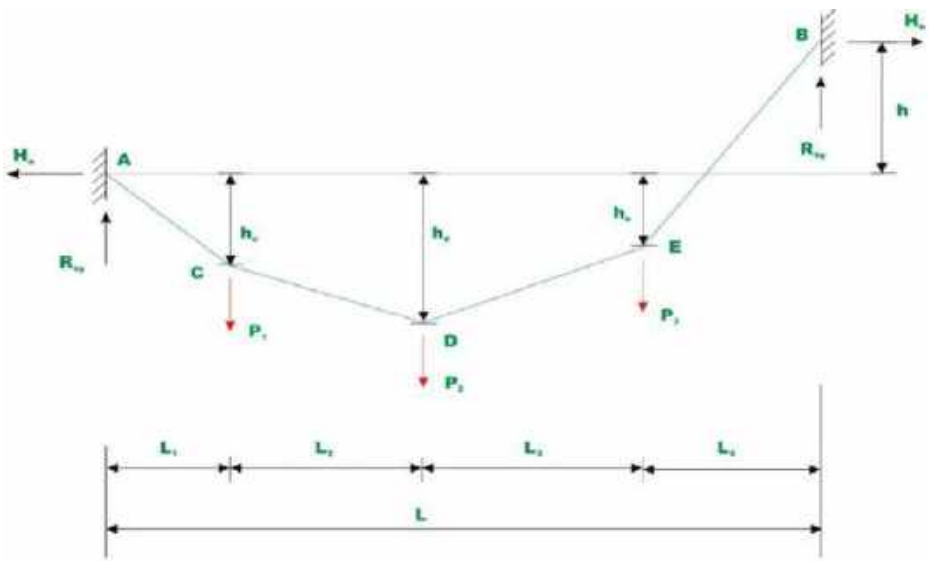


Fig-3

The shape assume... e or a chain (with no stiffnes s ... he action of external lo... hung from two supports is known as a funicular shape. Cable is a funicular structure. It is easy to vi... at a cable hung from two supports subjected to external load must be in tension. Now let us modify our definition of cable. **A cable may be defined as the structure in pure tension having the funicular shape of the load**.

Cable subjected to Concentrated Loads

As stated earlier, the cables are considered to be perfectly flexible (no flexural stiffness) and inextensible. As they are flexible they do not resist shear force and bending moment. It is subjected to axial tension only and it is always acting tangential to the cable at any point along the length. If the weight of the cable is negligible as compared with the externally applied loads then its self weight is neglected in the analysis. In the present analysis self weight is not considered.



Cable subjected to concentrated load.

Consider a cable as loaded in the Fig above. Let us assume that the cable lengths L_1, L_2, L_3, L_4 and sag at **C, D, E (h_c, h_d, h_e)** are known. The four reaction components at *A* and *B*, cable tensions in each of the four segments and three sag values: a total of eleven unknown quantities are to be determined. From the geometry, one could write two force equilibrium equations ($\sum F_x=0$, $\sum F_y=0$) at each of the point *A,B,C,D* and *E* i.e. a total of ten equations and the required one more equation may be written from the geometry of the cable. For example, if one of the sag is given then the problem can be solved easily. Otherwise if the total length of the cable ***S*** is given then the required equation may be written as

$$S = \sqrt{L_1^{\ 2} + h_c^{\ 2}} + \sqrt{L_2^{\ 2} + (h_d - h_c)^2} + \sqrt{L_2^{\ 2} + (h_d - h_e)^2} + \sqrt{L_2^{\ 2} + (h + h_e)^2}$$

Cable subjected to uniform load

Cables are used to support the dead weight and live loads of the bridge decks having long spans. The bridge decks are suspended from the cable using the hangers. The stiffened deck prevents the supporting cable from changing its shape by distributing the live load moving over it, for a longer length of cable. In such cases cable is assumed to be uniformly loaded.

Consider a cable which is uniformly loaded as shown in Fig 1. Let the slope of the cable be zero at *A*. Let us determine the shape of the cable subjected to uniformly distributed load. Consider a free body diagram of the cable as shown in Fig 31.3b. As the cable is uniformly loaded, the tension in the cable changes continuously along the cable length. Let the tension in the cable at ***m*** end of the free body diagram be ***T*** and tension at the ***n*** end of the cable be (**T+ΔT**). The slopes of the cable at **m** and **n** are denoted by **θ** and **θ+Δθ** respectively. Applying equations of equilibrium, we get

$$\sum Fy = 0 \qquad -T\sin\theta + (T+\Delta T)\sin(\theta+\Delta\theta) - q_0(\Delta x) = 0$$ ... ... *(Eq^n-1)*

$$\sum Fx = 0 \qquad -T\cos\theta + (T+\Delta T)\cos(\theta+\Delta\theta) = 0$$ ... ... *(Eq^n-2)*

$$\sum Mn = 0 \qquad -(T\cos\theta)\Delta y + (T\sin\theta)\Delta x + (q_0\Delta x)\frac{\Delta x}{2} = 0$$ ... ... *(Eq^n-3)*

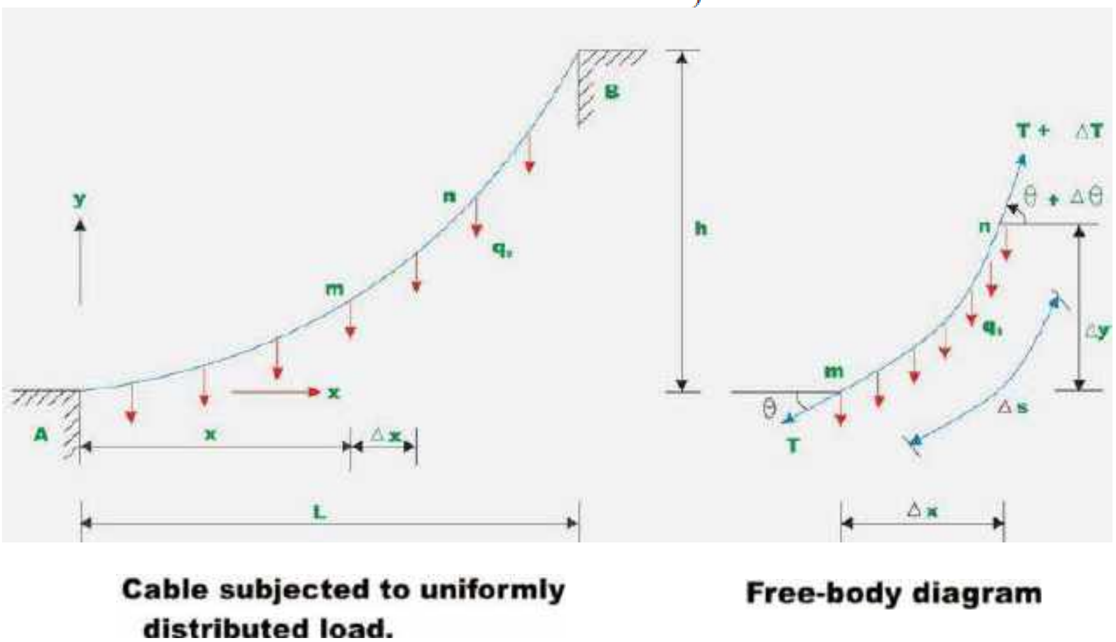


Fig-1 **Fig-2**

Dividing equations *Eq^n-1, 2 & 3* by Δx and noting that in the limit as Δx → 0, Δy→0, Δθ →0 and ΔT → 0;

$$\lim_{\Delta x \to 0} \quad \frac{\Delta T}{\Delta x}\sin(\theta+\Delta\theta) = q_0$$

$$\frac{d}{dx}(T\sin\theta) = q_0$$ ... ... *(Eq^n-4)*

$$\frac{d}{dx}(T\cos\theta) = 0$$ ... ... *(Eq^n-5)*

$$\lim_{\Delta x \to 0} \quad -T\cos\theta\frac{\Delta y}{\Delta x} +$$

$$\frac{dy}{dx} = \tan\theta \qquad T\cos\theta = \text{constant}$$

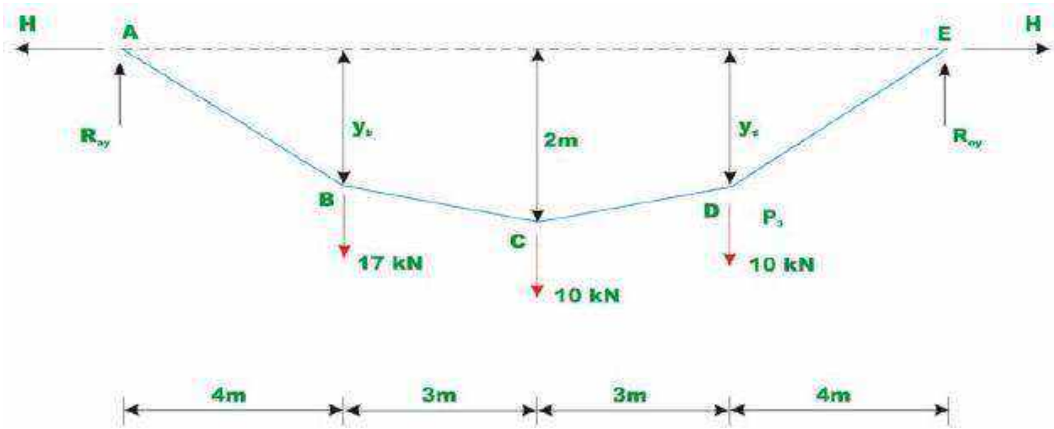
Integrating equation 5 we get,

At support (i.e., at x=0), **Tcosθ = H**

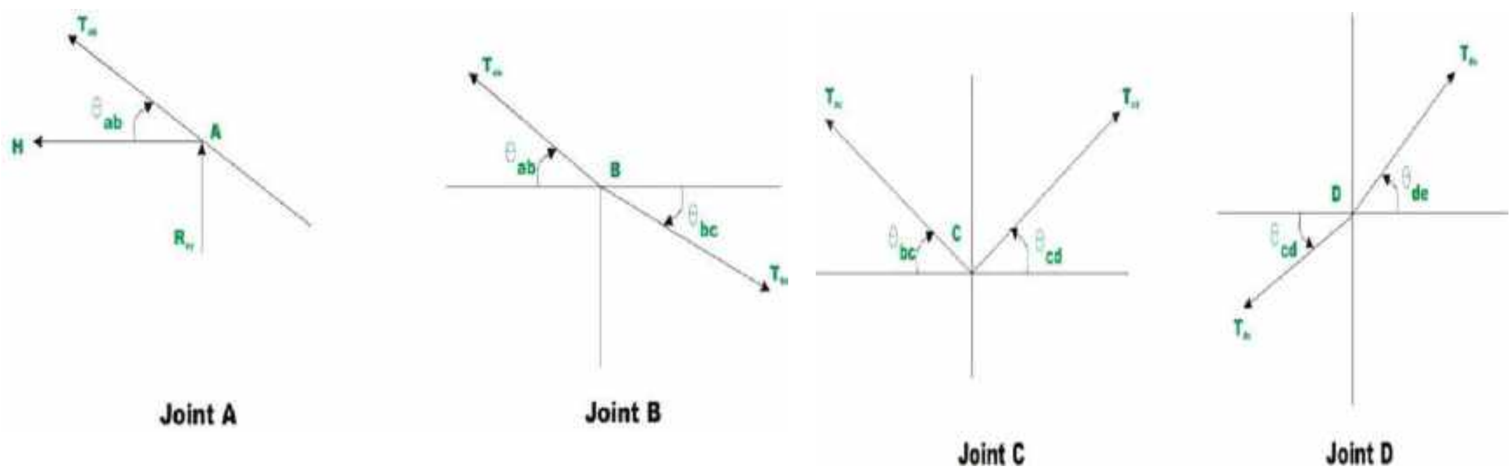
i.e. **horizontal component of the force along the length of the cable is constant.**

→Problem-1:

Determine reaction components at A and B, tension in the cable and the sag Y_B & Y_D of the cable shown in the following Fig. Neglect the self weight of the cable in the analysis.



Solution:



Since there are no horizontal loads, horizontal reactions at A and B should be the same. Taking moment about *E*, yields

$$R_{ay} \times 14 - 17 \times 20 - 10 \times 7 - 10 \times 4 = 0$$

$$R_{ay} = \frac{280}{14} = 20 \quad \text{kN}; \qquad R_{ey} = 37 - 20 = 17\text{kN}.$$

Now horizontal reaction *H* may be evaluated taking moment about point *C* of all forces left of *C*.

$$R_{ay} \times 7 - H \times 2 - 17 \times 3 = 0$$

$$H = 44.5 \text{ kN}$$

Taking moment about *B* of all the forces left of *B* and setting $\mathbf{M_B=0}$, we get

$$R_{ay} \times 4 - H \times y_B = 0; \qquad y_B = \frac{80}{44.50} = 1.798\,m$$

Similarly, $y_D = \dfrac{68}{44.50} = 1.528\,m$

To determine the tension in the cable in the segment *AB*, consider the equilibrium of joint *A*

$$\sum F_x = 0 \Rightarrow T_{ab}\cos\theta_{ab} = H$$

$$T_{ab} = \frac{44.5}{\left(3/\sqrt{3^2+0.298^2}\right)} = 48.789 \text{ kN}$$

The tension T_{ab} may also be obtained as,

$$T_{ab} = \sqrt{R_{ay}{}^2 + H^2} = \sqrt{20^2+44.5^2} = 48.789 \text{ kN}$$

Segment bc

Applying equations of equilibrium,

$$\sum F_x = 0 \Rightarrow T_{ab}\cos\theta_{ab} = T_{bc}\cos\theta_{bc}$$

$$T_{bc} = \frac{44.5}{\left(3/\sqrt{3^2+0.298^2}\right)} \cong 44.6 \text{ kN}$$

Segment cd

$$T_{cd} = \frac{T_{bc}\cos\theta_{bc}}{\cos\theta_{cd}} = \frac{44.5}{\left(3/\sqrt{3^2+0.472^2}\right)} = 45.05 \text{ kN}$$

Segment de

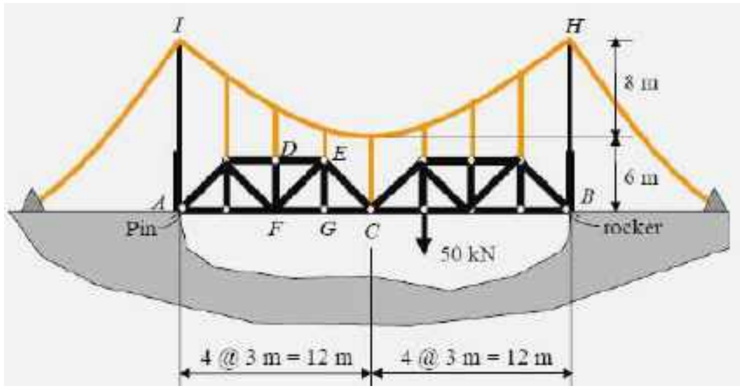
$$T_{de} = \frac{T_{cd}\cos\theta_{cd}}{\cos\theta_{de}} = \frac{44.5}{4/\sqrt{4^2+1.528^2}} = 47.636 \text{ kN}$$

The tension T_{de} may also be obtained as,

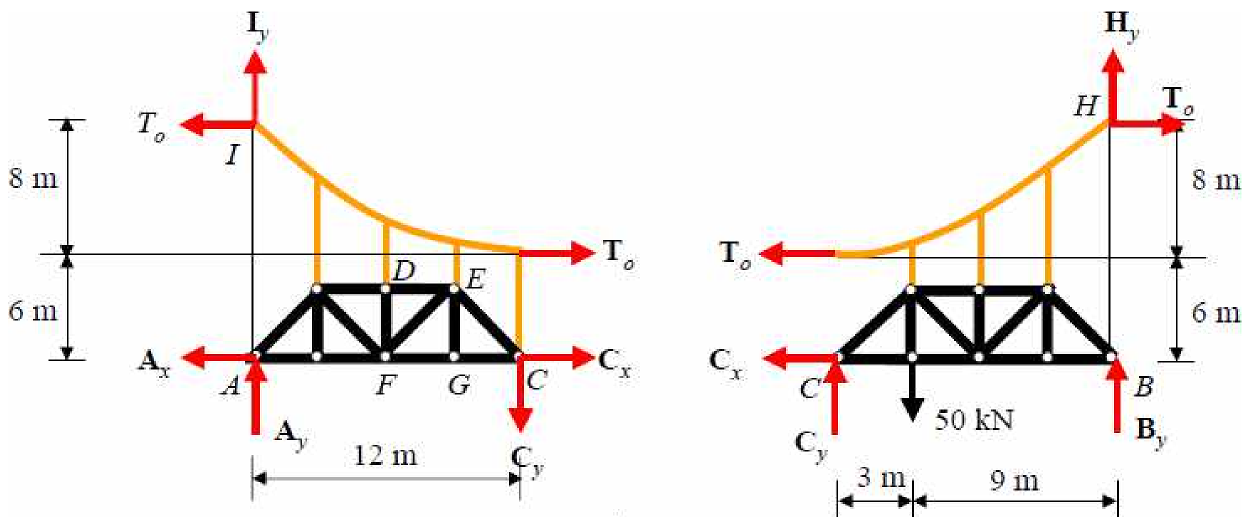
$$T_{de} = \sqrt{R_{ey}{}^2 + H^2} = \sqrt{17^2+44.5^2} = 47.636 \text{ kN}$$

→Problem-2:

The suspension bridge in the figure below is constructed using the two stiffening trusses that are pin connected at their ends C and supported by a pin at A and a rocker at B. Determine the maximum tension in the cable IH. The cable has a parabolic shape and the bridge is subjected to the single load of 50 kN.



Solution:



$\circlearrowright^{+} \; \Sigma M_A = 0$:

$$-12C_y + 8T_o = 0$$

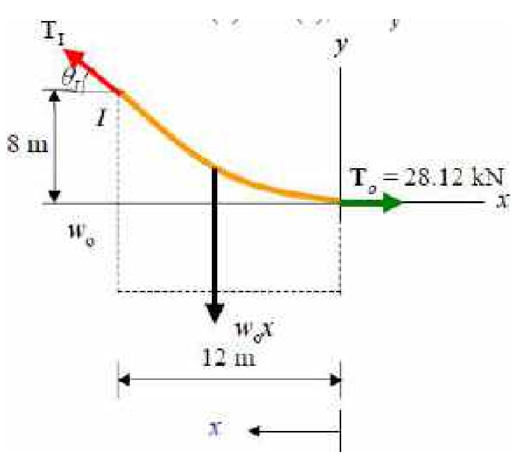
$$T_o = 1.5C_y \qquad \text{----------(1)}$$

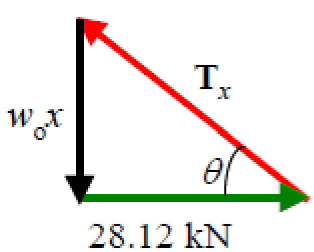
$\circlearrowright^{+} \; \Sigma M_B = 0$:

$$-12C_y + 50(9) - 8T_o = 0$$

$$T_o = -1.5C_y + 56.25 \qquad \text{----------(2)}$$

From (1) and (2), $\quad C_y = 18.75$ kN, $\quad T_o = 28.125$ kN





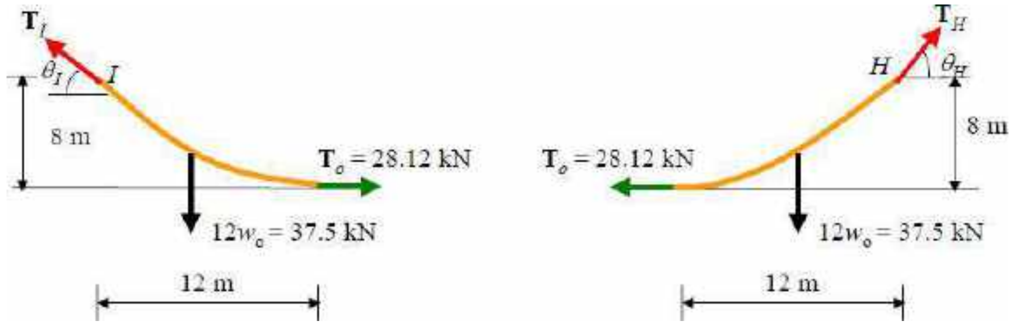
$$\frac{dy}{dx} = \tan\theta = \frac{w_o x}{28.12}$$

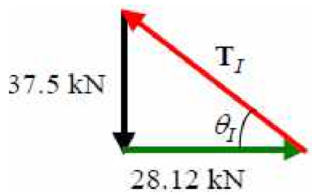
$$y = \int \frac{w_o x}{28.12} dx$$

$$y = \frac{w_o x^2}{28.12} + \cancel{C_1}^{\,0}$$

$$8 = \frac{w_o (12)^2}{2(28.12)}$$

$$w_o = 3.125 \text{ kN/m}$$



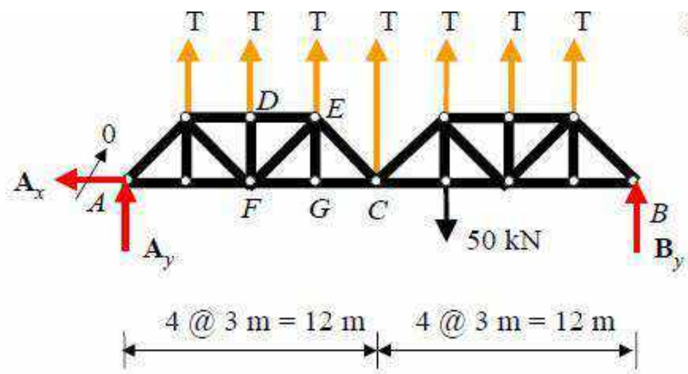


$$T_I = \sqrt{(37.5)^2 + (28.12)^2}$$

$$= 46.88 \text{ kN}$$

$$T_{\max} = T_I = T_H = 46.88 \text{ kN}$$

$$T_{\min} = T_o = 28.12 \text{ kN}$$



$$T = w_o \times 3 = 3.125 \times 3$$

$$= 9.375 \text{ kN}$$

$\circlearrowright + \; \Sigma M_A = 0: \quad 9.375(3+6+9+12+15+18+21) - 50(15) + B_y(24) = 0$

$$B_y = \text{-}1.56 \text{ kN}, \downarrow$$

$+\uparrow \; \Sigma F_y = 0: \quad A_y + 7(9.375) - 50 - 1.56 = 0$

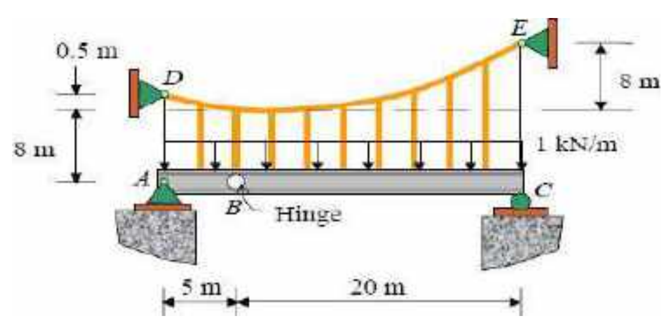
$$A_y = \text{-}14.07 \text{ kN}, \downarrow$$

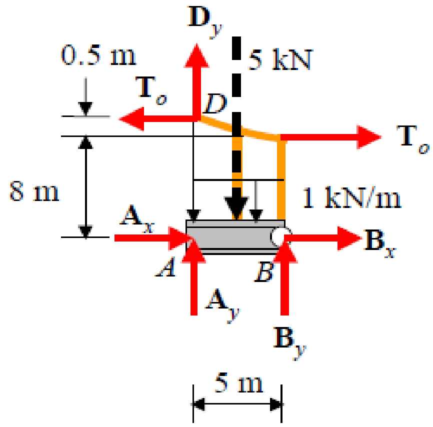
→Problem-3:

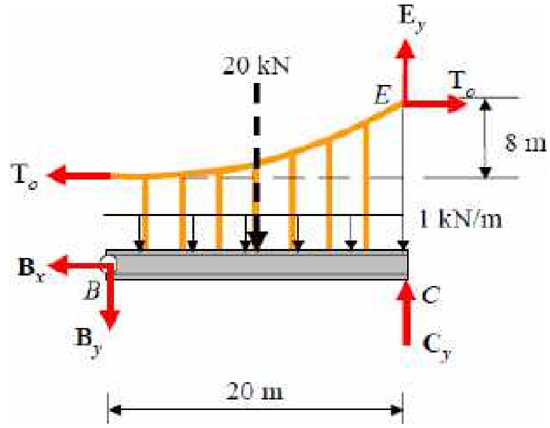
For the structure shown:

(a) Determine the **maximum tension** of the cable

(b) Draw **quantitative shear & bending-moment diagrams** of the beam.







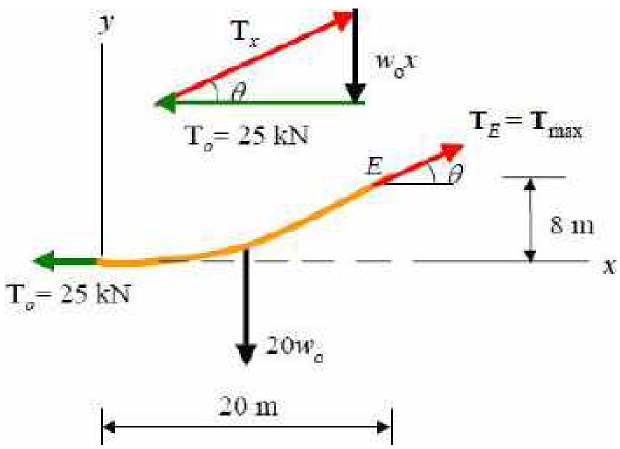
$+\circlearrowright \Sigma M_A = 0$:

$$B_y(5) - 5(2.5) + T_o(0.5) = 0$$

$+\circlearrowright \Sigma M_C = 0$:

$$B_y(20) + 20(10) - T_o(8) = 0$$

From (1) and (2), $B_y = 0$, $T_o = 25$ kN



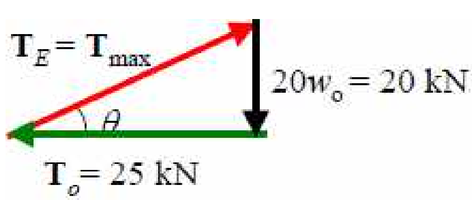
$$\frac{dy}{dx} = \tan\theta = \frac{w_o x}{25}$$

$$y = \int \frac{w_o x}{25}\,dx$$

$$= \frac{w_o x^2}{2(25)} + \cancel{C_1}^{\,0}$$

$$8 = \frac{w_o(20)^2}{2(25)}$$

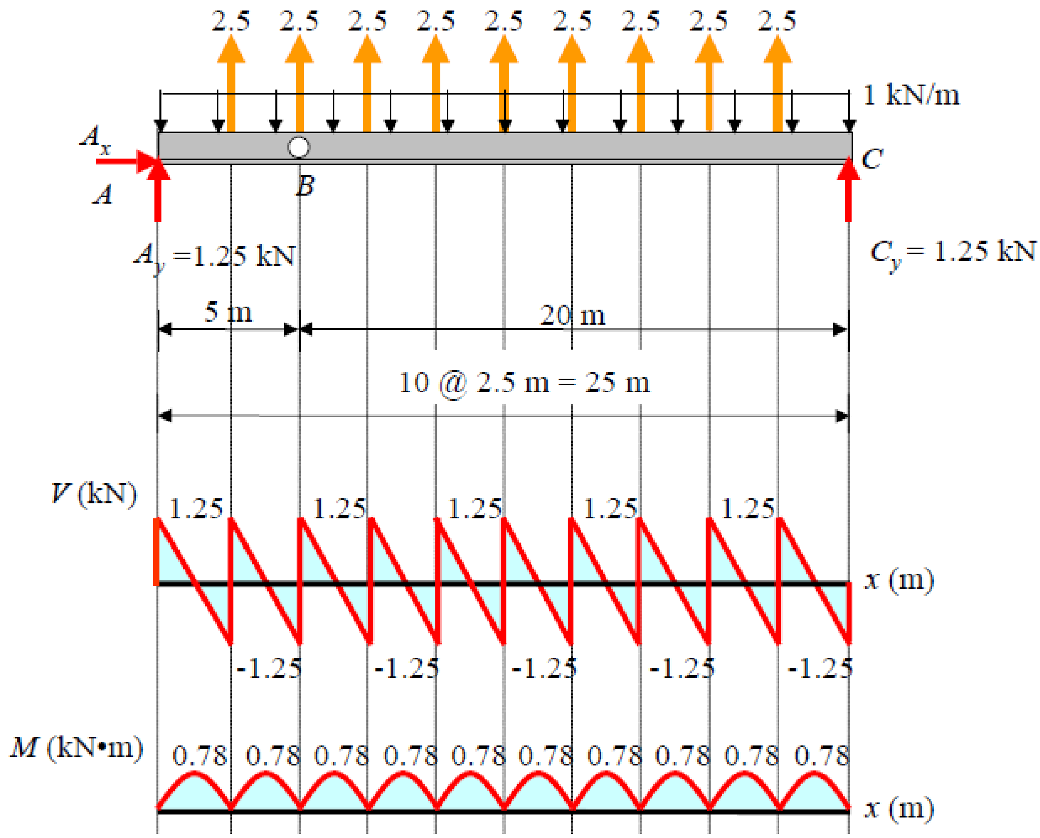
$$w_o = 1 \text{ kN/m}$$



$$T_{max} = T_E = \sqrt{(25)^2 + (20)^2}$$

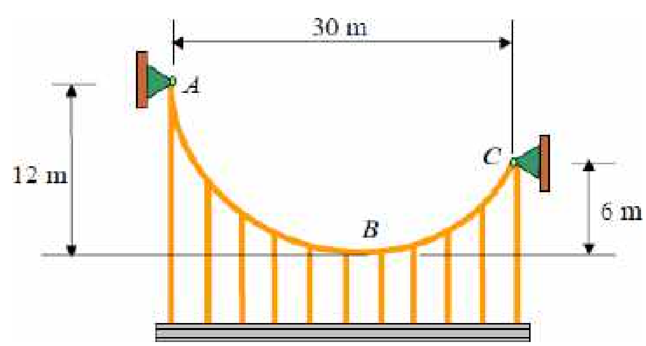
$$T_{max} = 32.02 \text{ kN}$$

$T = w_o(2.5 \text{ m}) = (1\text{kN/m})(2.5 \text{ m}) = 2.5 \text{ kN}$

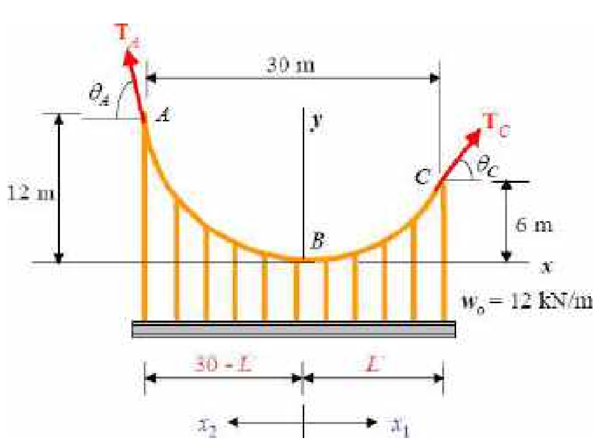


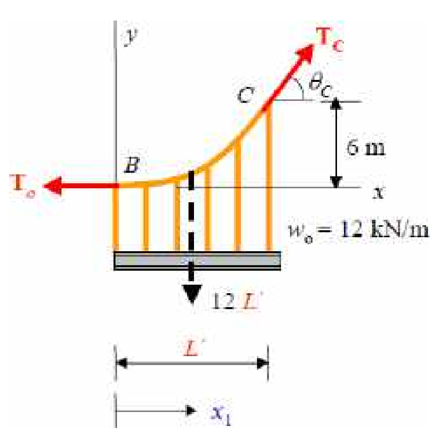
→Problem-4

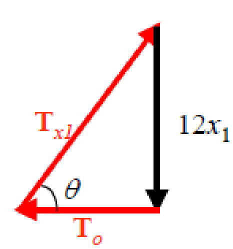
The cable shown supports a girder which weighs 12kN/m. Determine the tension in the cable at points *A*, *B*, and *C*.



Solution:







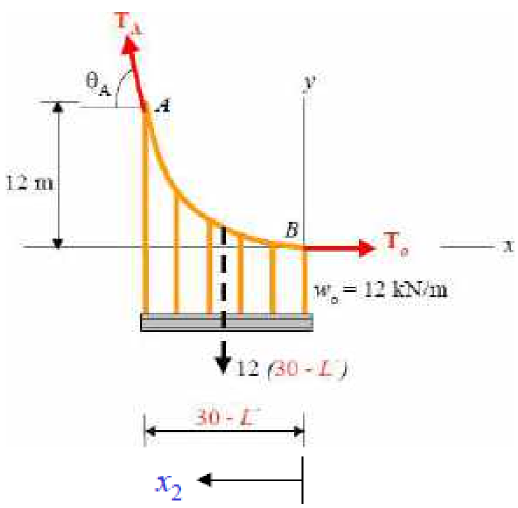
$$\frac{dy_1}{dx_1} = \tan\theta = \frac{12x_1}{T_o}$$

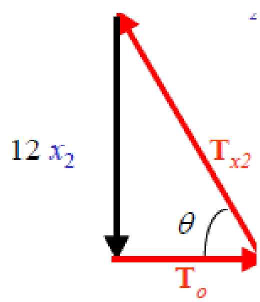
$$y_1 = \int \frac{12x_1}{T_o}\,dx_1$$

$$y_1 = \frac{12x_1^2}{2T_o} + \cancel{C_1}^{\,0}$$

$$6 = \frac{12L'^2}{2T_o}$$

$$\boxed{T_o = L'^2} \qquad \text{----------(1)}$$





$$\frac{dy_2}{dx_2} = \tan\theta = \frac{12x_2}{T_o}$$

$$y_2 = \int \frac{12x_2}{T_o}\,dx_2 = \frac{12x_2^2}{2T_o} + \cancel{C_2}^{\,0}$$

$$y_2 = \frac{12x_2^{\,2}}{2T_o}$$

$$\cancel{1}2 = \frac{\cancel{1}2(30-L')^2}{2T_o}$$

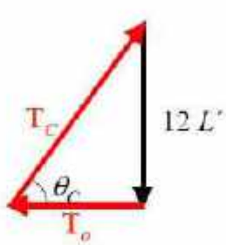
$$\boxed{1 = \frac{(30-L')^2}{2T_o}} \qquad \text{----------(2)}$$

$$T_o = L'^2 \qquad \text{----------(1)}$$

$$1 = \frac{(30-L')^2}{2T_o} \qquad \text{----------(2)}$$

From (1) and (2), $L' = 12.43$ m, $T_o = 154.5$ kN

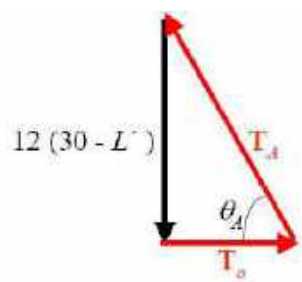
$$T_B = T_o = 154.5 \text{ kN}$$



$$T_C = \sqrt{T_o^{\,2} + (12L')^2}$$

$$= \sqrt{(154.50)^2 + (12\times 12.43)^2}$$

$$= 214.8 \text{ kN}$$



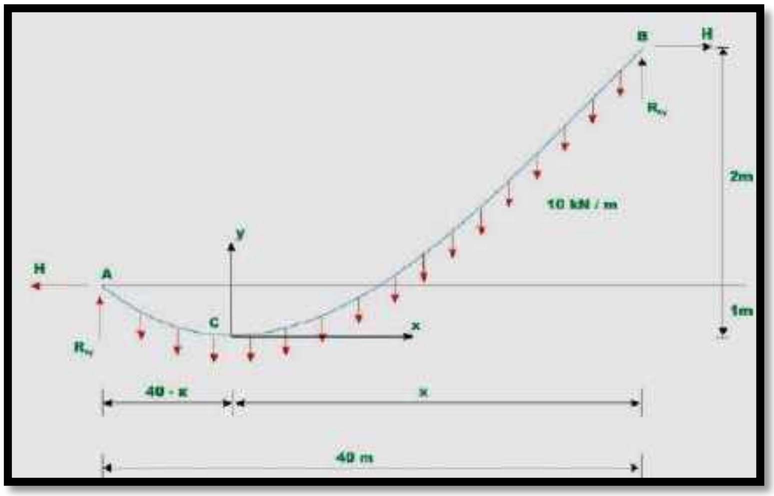
$$T_A = \sqrt{T_o^{\,2} + [12(30-L')]^2}$$

$$= \sqrt{(154.50)^2 + [12(30-12.43)]^2}$$

$$= 261.4 \text{ kN}$$

→Problem-5:

A cable of uniform cross section is used to span a distance of 40m as shown in Fig. The cable is subjected to uniformly distributed load of 10 kN/m. run. The left support is below the right support by 2 m and the lowest point on the cable *C* is located below left support by 1 m. Evaluate the reactions and the maximum and minimum values of tension in the cable.



Solution:

Assume the lowest point *C* to be at distance of *x* m from *B*. Let us place our origin of the co-ordinate system *xy* at C.

$$y_a = 1 = \frac{q_0(40-x)^2}{2H} = \frac{10(40-x)^2}{2H} \qquad \text{... ... ... (Eq}^n\text{-1)}$$

$$y_b = 3 = \frac{10x^2}{2H} \qquad \text{... ... ... (Eq}^n\text{-2)}$$

Where y_a and y_b be the co-ordinates of supports *A* and *B* respectively. From equations 1 and 2, one could evaluate the value of *x*.

$$10(40-x)^2 = \frac{10x^2}{3} \quad \Rightarrow \quad x = 25.359\,m$$

From equation 2, the horizontal reaction can be determined.

$$H = \frac{10 \times 25.359^2}{6} = 1071.80 \text{ kN}$$

Now taking moment about *A* of all the forces acting on the cable, yields

$$R_{by} = \frac{10 \times 40 \times 20 + 1071.80 \times 2}{40} = 253.59 \text{ kN}$$

Writing equation of moment equilibrium at point *B*, yields

$$R_{ay} = \frac{40 \times 20 \times 10 - 1071.80 \times 2}{40} = 146.41 \text{ kN}$$

Tension in the cable at supports *A* and *B* are

$$T_A = \sqrt{146.41^2 + 1071.81^2} = 1081.76 \text{ kN}$$

$$T_B = \sqrt{253.59^2 + 1071.81^2} = 1101.40 \text{ kN}$$

The tension in the cable is maximum where the slope is maximum as $\boldsymbol{T \cos\Theta = H}$. The maximum cable tension occurs at *B* and the minimum cable tension occurs at *C* where $\frac{dy}{dx} = \Theta = 0$; and $T_c = H = 1071.81$ kN.

ADDITIONAL CONSIDERATIONS FOR CABLE SUPPORTED STRUCTURES

- **Forces on cable bridges**: Wind drag and lift forces - Aero-elastic effects should be considered (vortex-induced oscillations, flutter, torsional divergence or lateral buckling, galloping and buffeting).

- **Wind tunnel tests**: To examine the aerodynamic behavior

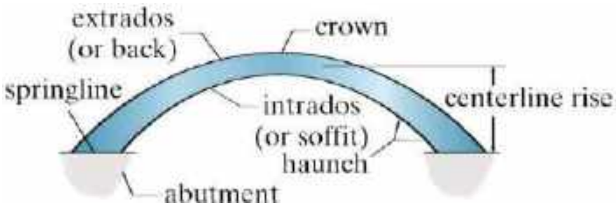
- **Precaution to be taken against**: Torsional divergence or lateral buckling due to twist in bridge; Aero-elastic stability caused by geometry of deck, frequencies of vibration and mechanical damping present; Galloping due to self-excited oscillations; Buffeting due to unsteady loading caused by velocity fluctuations in the wind flow

ARCHES

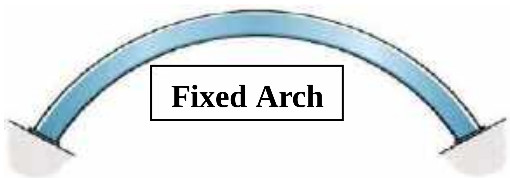
Arches can be used to reduce the bending moments in long-span structures. Essentially, an arch acts as an inverted cable, so it receives its load mainly in compression although, because of its rigidity, it must also resist some bending and shear depending upon how it is loaded and shaped.

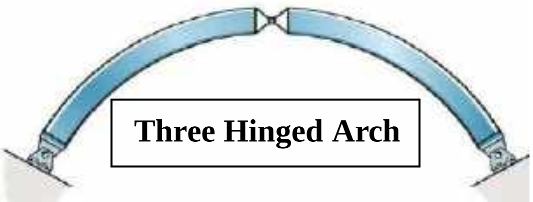
In particular, if the arch has a parabolic shape and it is subjected to a uniform horizontally distributed vertical load, then only compressive forces will be resisted by the arch. Under these conditions the arch shape is called a funicular arch because no bending or shear forces occur within the arch.

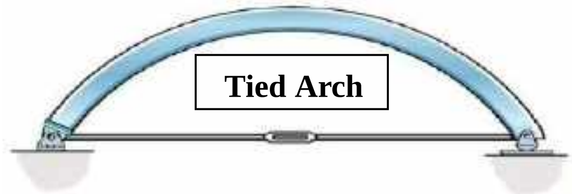
Different terms and types of Arches



Depending on its uses, several types of arches can be selected to support a loading:







Fixed Arch:

A *fixed arch* is often made from reinforced concrete. Although it may require less material to construct than other types of arches, it must have solid foundation abutments since it is indeterminate to the third degree and, consequently, additional stresses can be introduced into the arch due to relative settlement of its supports.

Two Hinged Arch:

A *two-hinged arch* is commonly made from metal or timber. It is indeterminate to the first degree, and although it is not as rigid as a fixed arch, it is somewhat insensitive to settlement. We could make this structure statically determinate by replacing one of the hinges with a roller. Doing so, however, would remove the capacity of the structure to resist bending along its span, and as a result it would serve as a curved beam, and not as an arch.

Three Hinged Arch:

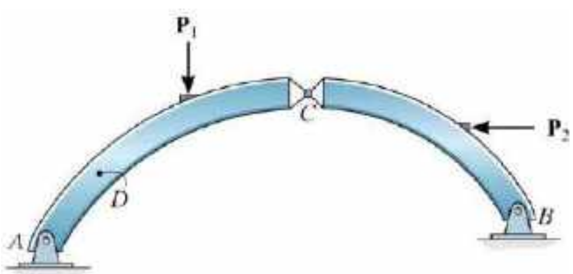
A *three-hinged arch* which is also made from metal or timber, is statically determinate. Unlike statically indeterminate arches, it is not affected by settlement or temperature changes.

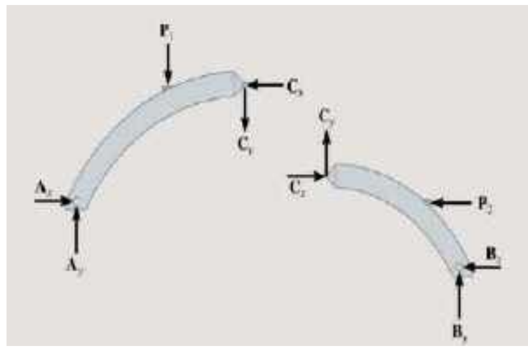
Tied Arch:

If two and three-hinged arches are to be constructed without the need for larger foundation abutments and if clearance is not a problem, then the supports can be connected with a tie rod. A tied arch allows the structure to behave as a rigid unit, since the tie rod carries the horizontal component of thrust at the supports. It is also unaffected by relative settlement of the supports.

Three-Hinged Arches

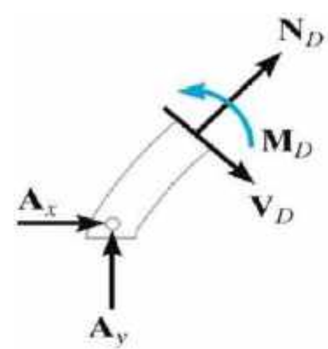
- ✓ The third hinge is located at the crown & the supports are located at different elevations
- ✓ To determine the reactions at the supports, the arch is disassembled





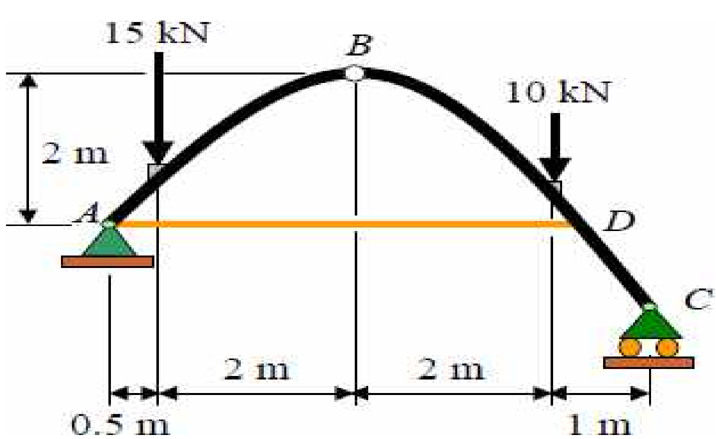
In order to determine the reactions at the supports, the arch is disassembled and the free-body diagram of each member. Here there are six unknowns for which six equations of equilibrium are available. One method of solving this problem is to apply the moment equilibrium equations about points *A* and *B*. Simultaneous solution will yield the reactions C_x and C_y. The support reactions are then determined from the force equations of equilibrium.

Once all support reactions obtained, the internal normal force, shear, and moment loadings at any point along the arch can be found using the method of sections. Here, of course, the section should be taken perpendicular to the axis of the arch at the point considered.



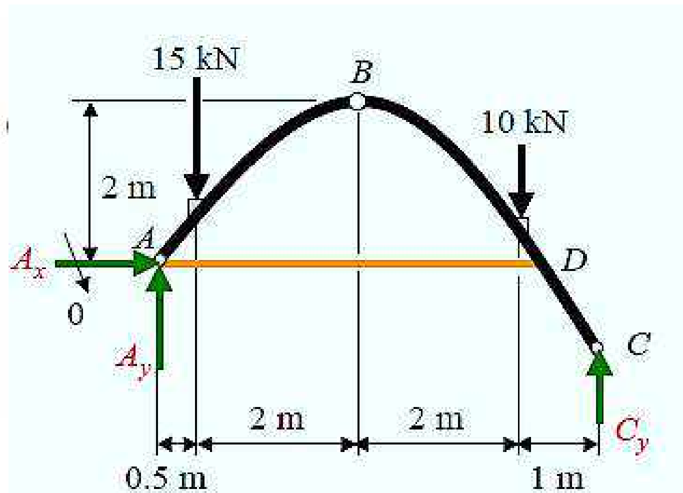
→Problem-6:

The tied three-hinged arch is subjected to the loading shown. Determine the components of reaction at A and C and the tension in the cable.



Solution:

Entire arch :



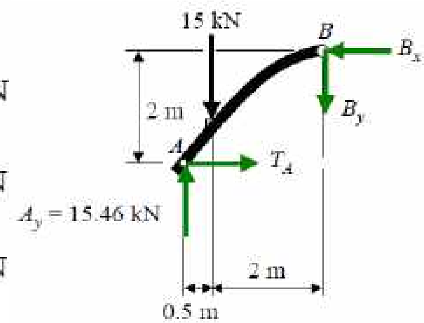
$\circlearrowright + \ \Sigma M_A = 0$: $\quad C_y(5.5) - 10(4.5) - 15(0.5) = 0$

$C_y = 9.545$ kN

$+\uparrow \ \Sigma F_y = 0$: $\quad A_y - 15 - 10 + 9.545 = 0$

$A_y = 15.46$ kN

Member *AB* :



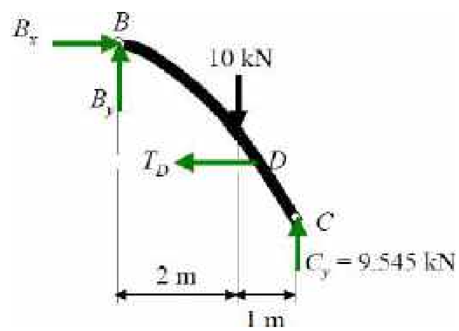
$\circlearrowright + \ \Sigma M_B = 0$: $\quad 15(2) - 15.455(2.5) + T_A(2) = 0 \qquad T_A = 4.319$ kN

$+\uparrow \ \Sigma F_y = 0$: $\quad 15.455 - 15 - B_y = 0 \qquad B_y = 0.455$ kN

$\xrightarrow{+} \ \Sigma F_x = 0$: $\quad 4.319 - B_x = 0 \qquad B_x = 4.319$ kN

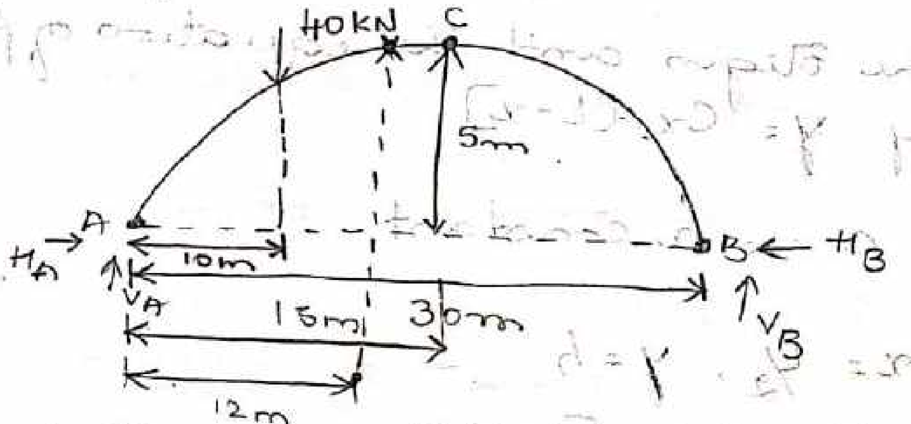
Member *AB* :

$\xrightarrow{+} \ \Sigma F_x = 0$: $\quad 4.319 - T_D = 0 \qquad T_D = 4.319$ kN



Problems:

1. A 3 hinged Symmetrical parabolic arch has a span of 30m and Central rise 5m subjected to a point load 40KN at 10m from left hinge draw Bending moment diagrams and Calculate reactn and also normal thrust & Radial shear at 12m from left hinge



step: Calculation of reaction

$\Sigma V = 0$

$-40 + V_A + V_B = 0$

$V_A + V_B = 40\ kN$

$\Sigma H = 0$

$H_A - H_B = 0$

$H_A = H_B$

$\Sigma M_A = 0$

$(H_A \times 0) + (V_A \times 0) + (H_B \times 0) + (40 \times 10) - (V_B \times 30) = 0$

$V_B = \frac{40\cancel{0}}{3\cancel{0}}$

$\boxed{V_B = 13.33\ KN}$

$V_A = 40 - 13.33$

$\boxed{V_A = 26.68\ KN}$

$\Sigma M_C = 0$ [Half portn is sufficient).

$-(40 \times 5) - (H_A \times 5) + V_A \times 15 = 0.$

$H_A \times 5 = 26.66 \times 15 - 40 \times 5$

$\boxed{H_A = \frac{199.9}{5}}$

$H_A = 39.98$

$\boxed{H_A = 40\ KN}$ $\boxed{H_B = 40\ KN}$

Step 2 : ~~Computation~~ Computation of Vertical ordinate and slope eq^n.

$$Y = \frac{4hx}{l^2}[l-x]$$

$$= \frac{4(5)(x)}{30^2}[30-x]$$

$$= \frac{20}{50^2}[30x-x^2]$$

$$= 0.022[30x-x^2]$$

$$\boxed{Y = 0.66x - 0.022x^2}$$

$$\tan\theta = \frac{dy}{dx}$$

$$= \frac{d}{dx}[0.66x - 0.022x^2]$$

$$= 0.66 - 2(0.022)x$$

$$\boxed{\tan\theta = 0.66 - 0.044x}$$

Step 3 : Calculation of Normal thrust and Radial Shear @ 12 m from left hinge

$x = 12m$

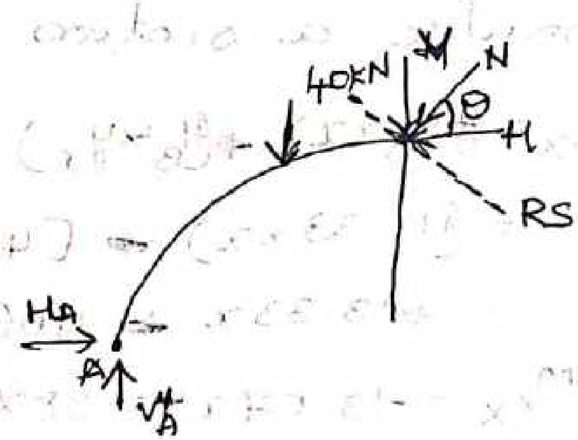
$\tan\theta = 0.66 - 0.044(12)$

$\tan\theta = 0.132$

$\theta = \tan^{-1}(0.132)$

$\boxed{\theta = 7.51}$ rad

$\theta =$



$V = V_A - 40$

$= 26.66 - 40$

$V = -13.34 kN$

$\boxed{H = H_A = 40 kN}$

Normal thrust $= H\cos\theta + V\sin\theta$

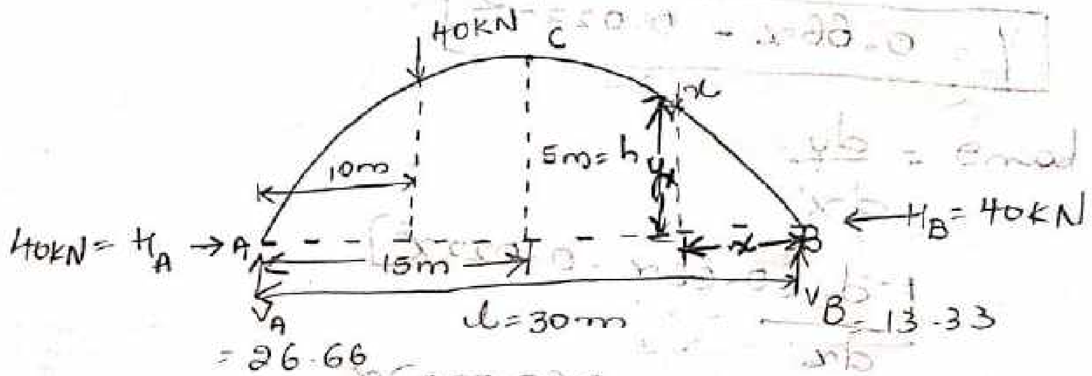
$= 40\cos[7.51] + -13.34\sin(7.51)$

$= 37.91\,kN$

Radial shear $= H\sin\theta - V\cos\theta$

$= 40\sin(7.51) - (-13.34)\cos(7.51)$

$= 18.45\,kN$



Step3: Calculation of Bending moment

$M_A = M_B = M_C = 0$ [There will be 0 moment at hinges]

Take M_D

$(V_A \times 10) - (H_A \times (0.66(10) - 0.022(10)^2))$

$= (26.66 \times 10) - [40 \times [0.66(10) - 0.022(10)^2]]$

$= 90.6\,kN\text{-}m$

Consider a section xx at a distance of x from B

$M_{xx} = (-V_B \times x) + (H_B \times y_x)$

$= (-13.33 \times x) + [40 \times [0.66x - 0.022x^2]]$

$= -13.33x + 40(0.66)x - 4(0.022)x^2$

$M_{xx} = -13.07x + 0.88x^2$

B/w two hinged point [C & B] there is no load so we will get the maximum bending moment equate

$$\frac{dM_{xx}}{dx} = 0$$

$$\frac{d}{dx}\left[-13.07x + 0.88x^2\right] = 0$$

$$-13.07 + 2(0.88)x = 0$$

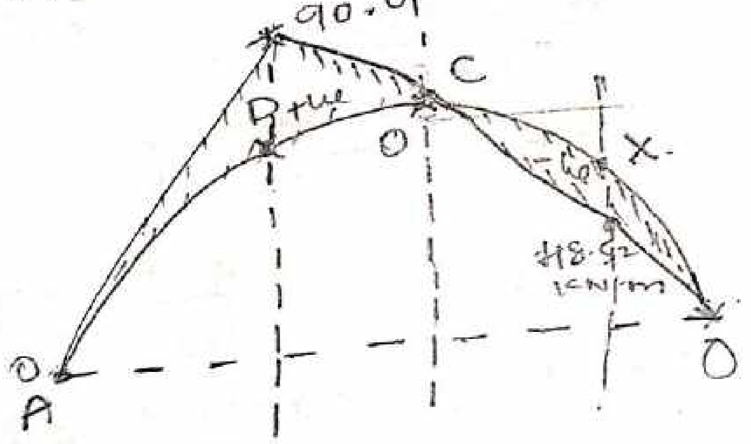
$$2(0.88)x = 13.07$$

$$x = \frac{13.07}{1.76}$$

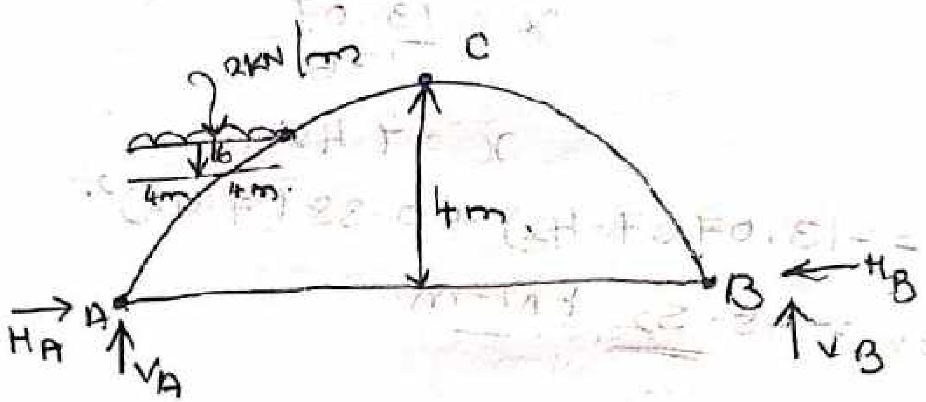
$$x = 7.42\,m$$

$$M_{xx} = -13.07(7-42) + 0.88(7-42)^2$$

$$M_{xx} = -48.82\ kN\text{-}m$$



A three hinged parabolic arch has a span of 20m and rise 4m. It is loaded with UDL of intensity 2kN/m on left 8m length. Find direction and magnitude of reaction at the hinges A & B and also find out max +ve & -ve Bending moment.



$\Sigma H = 0$

$H_A - H_B = 0$

$\boxed{H_A = H_B}$

$V_A + V_B - 16 = 0$

$\boxed{V_A + V_B = 16}$

$\Sigma M_A = 0.$

$(H_A \times 0) + (V_A \times 0) + (H_B \times 0) - (V_B \times 20) + (H_B \times 0) + (16 \times 4) = 0$

$V_B = 3.2\,kN.$

$\boxed{V_A = 12.8\,KN}$

$\Sigma M_C = 0$

$-(16 \times 6) + (12.8 \times 10) - (H_A \times 4) = 0$

$-(H_A \times 4) = -32$

$H_A = +\frac{32}{4}$

$\boxed{H_A = 8}$

$\boxed{H_A = H_B = 8\,kN.}$

computation of vertical ordinate and slope

$R_A = \sqrt{V_A^2 + H_A^2}$

$= \sqrt{(12.8)^2 + (8)^2}$

$= 15.094 kN$

$R_B = \sqrt{V_B^2 + H_B^2}$

$= \sqrt{(3.2)^2 + (8)^2}$

$= 8.61$

$\theta_A = \tan^{-1}\left[\frac{V_A}{H_A}\right]$

$= \tan^{-1}\left[\frac{12.8}{8}\right]$

$= 57.99$

$\theta_B = \tan^{-1}\left[\frac{V_B}{H_B}\right]$

$= \tan^{-1}\left[\frac{3.2}{8}\right]$

$\theta_B = 21.80$

calculation of BM.

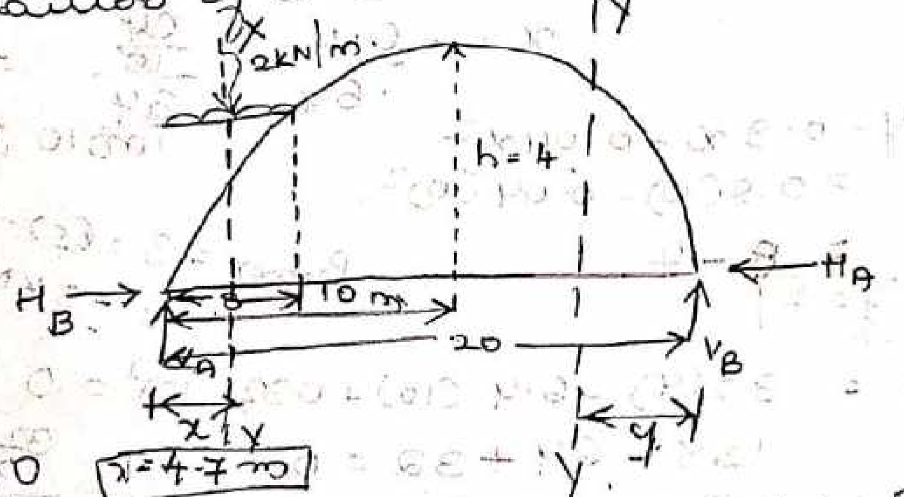
$y = \frac{4hx}{l^2}(l-x)$

$y = \frac{4 \times 4 \times x}{20^2}(20-x)$

$$\boxed{y = 0.8x - 0.04x^2}$$

maximum the bending moment may occur somewhere under the UDL.

and max negative B M to occur somewhere in the position of B.C



$B_{mA} = B_{mB} = BM_C = 0$

$BM_{XY} = 0$ $\boxed{x = 4.7m}$

$(V_A \times x) - (H_A \times y) = 0$

$12.8x - 8(0.8x - 0.04x^2) = 0$

$12.8x - 6.4x + 0.32x^2 = 0$

$0.32x^2 + 6.4x = 0$

$0.32x^2 + 6.4x (x \times 2 \times x/x)$

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$\Sigma M_C = 0$ [Consider half of portion) Right

$(75 \times 6) + (H_B \times 4) - (V_B \times 12) = 0$

$H_B \times 4 = (191.25 \times 12) - (75 \times 6)$

$H_B = \frac{1845}{4}$

$\boxed{H_B = 461.25 \text{ kN-m}}$.

$V_A + V_B = 615 - 191.25$.

$V_A = 423.75$.

Step 2: Computation of vertical ordinate eqn & slope eqn

$y = \frac{4hx}{l^2}[l - x]$.

$y = \frac{4(4)x}{(24)^2}(24 - x]$;

$y = 0.027x[24 - x]$;

$\boxed{y = 0.648x - 0.027x^2}$

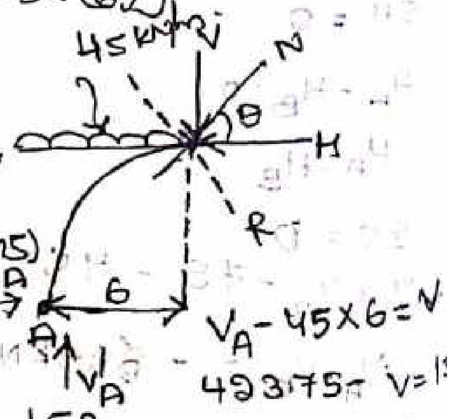
$\tan\theta = \frac{dy}{dx}$

$= \frac{d}{dx}[0.648x - 0.027x^2]$.

$\tan\theta = 0.648 - 0.054x$.

$\theta = \tan^{-1}(0.648 - 0.054(6))$;

$\theta = 17.95$



Normal thrust = $H\cos\theta + V\sin\theta$

$= 461.25\cos(17.95) + \cancel{423.75}\ 153.75 \sin(17.95)$.

$= \cancel{585.06}$ kN // 485.94 kN

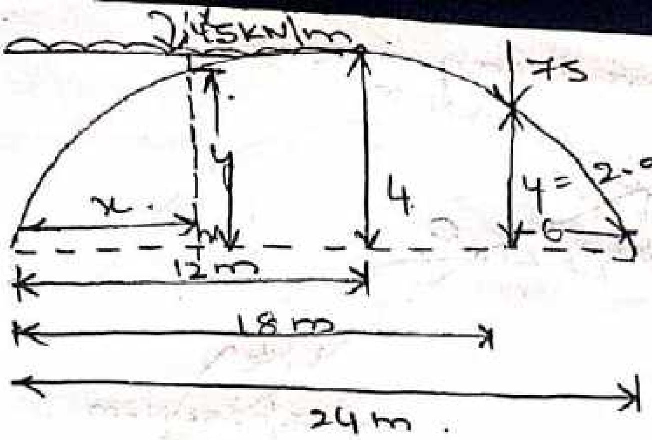
$V_A - 45 \times 6 = V$

$423.75 = V = 153.$

Radial shear $= H\sin\theta - V\cos\theta$

$= 461.25\sin(17.95) - \cancel{423.75}\ 153.75\cos(17.95)$

$= \cancel{545.27}$ kN // $= -4.1921$ kN



$BM_{xx} = [V_A \times x] - (H_A \times y) - (45 \times x)\, x/2 = 0$

$BM_{xx} = 483.75x - 461.25[0.648x - 0.027x^2] - 22.5x^2 = 0$

$= 483.75x - 298.89x + 12.45x^2 - 22.5x^2$

$BM_{xx} = 124.86x - 10.04x^2$

To get maximum Bending moment diff Moment eqn w.r.t x

$\frac{d}{dx}[BM_{xx}] = 0$

$\frac{d}{dx}[124.86x - 10.04x^2] = 0$

$124.86 - 20.08x = 0$

$124.86 = 20.08x$

$x = \frac{124.86}{20.08}$

$\boxed{x = 6.21m}$

$BM_{xx} = 124.86x - 10.04x^2$

$= 124.86[6.21] - 10.04[6.21]^2$

$\boxed{BM_{xy} = 388.19\ kN\text{-}m}$

from. Ordinate eqn

$y = 0.648x - 0.027x^2$

$= 0.648(6) - 0.027(6)^2$

$y = 2.91m$

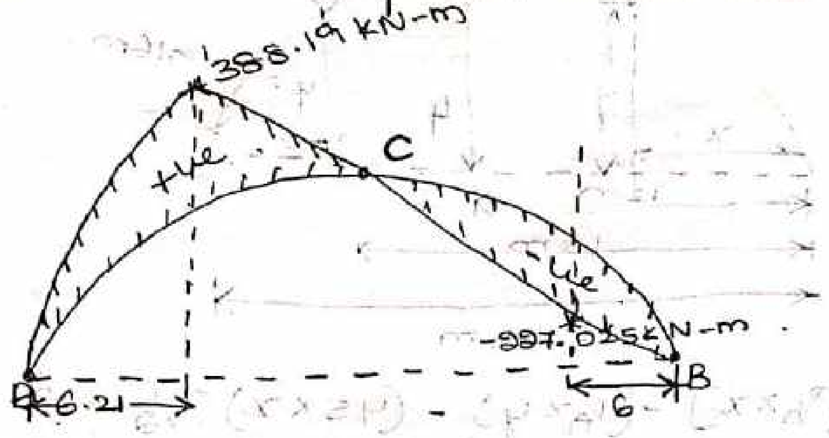
$BM_D = (V_B \times 6) - (H_B \times 2.98)$

$= (191.25 \times 6) - (461.25 \times 2.98)$

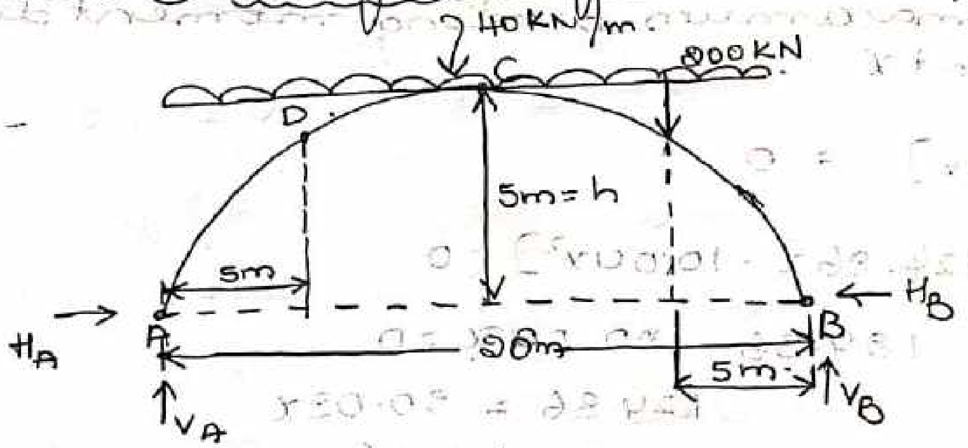
$\boxed{BM_D = -227.025\ KNm}$

$\curvearrowleft | \uparrow +y$
$\downarrow | \downarrow +ve$

Bending moment Diagram



A 3 hinged parabolic arch of 20m symmetrical span 5m rise carries UDL of 40KN/m on the entire span and a point load of 200KN @ 5m from right end. Determine bending moment, Normal thrust and radial shear @ 5m from left end.



$\Sigma H = 0$

$H_A - H_B = 0$

$H_A = H_B$

$\Sigma V = 0$

$V_A + V_B - (40 \times 20) - 200 = 0$

$V_A + V_B = 1000\,KN$

$\Sigma M_A = 0$

$(H_A \times 0) + V_A \times 0) + (40 \times 20)\frac{20}{2} + 200 \times 15 + (H_B \times 0) - V_B \times 20 = 0$

$V_B \times 20 = 11000$

$V_B = 11000/20$

$V_B = 550\,KN$

$V_A = 1000 - 550$

$\boxed{V_A = 450\,KN}$

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ΣM_C considers half of the sect^n left

$\Sigma M_C = -(H_A \times 5) + (V_A \times 10) - (40 \times 10)5$ ↻ +ve

$= -H_A \times 5 + 450 \times 10 - [40 \times 100]$

$\boxed{H_A = 500\,kN = H_B}$

$y = \frac{4hx}{l^2}(l-x)$

$= \frac{4 \times 5 \times x}{20^2}(20-x)$

$= \frac{x}{20}(20-x)$.

$y = x - 0.05x^2$

$y = (5) - 0.05(5)^2$.

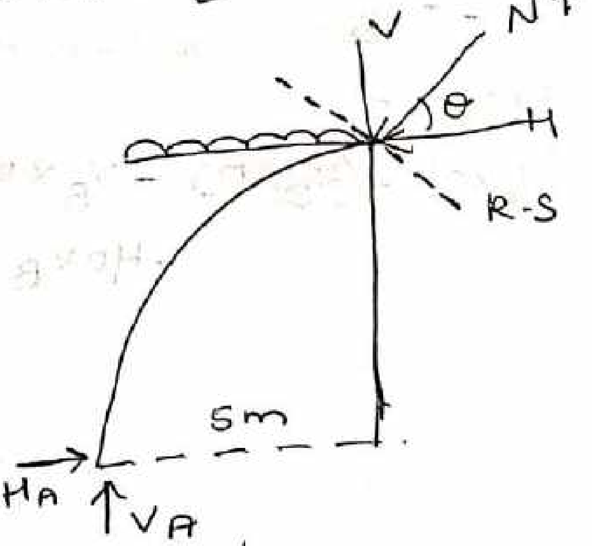
$\boxed{y = 3.75\,m}$.

$(40 \times 5)(5/2) - [H_A \times 3.75] + (V_A \times 5) = 0$

$BM_D = (40 \times 5)(5/2) - [H_A \times 3.75] + [450 \times 5]$

$= (40 \times 5)(2.5) - (500 \times 3.75) + [450 \times 5]$

$= -125\,kN \cdot m$



$V = (-40 \times 5) + 450$

$= 250$

$H = H_A = 500\,kN$.

Normal thr $= H\cos\theta + V\sin\theta$

$= 500\cos(26.56) + 250\sin(26.56)$

$= 559.01\,kN$

$\frac{dy}{dx} = \tan\theta$

$1 - 0.1(5) = \tan\theta$

$\tan\theta = 0.5$

$\theta = \tan^{-1}$

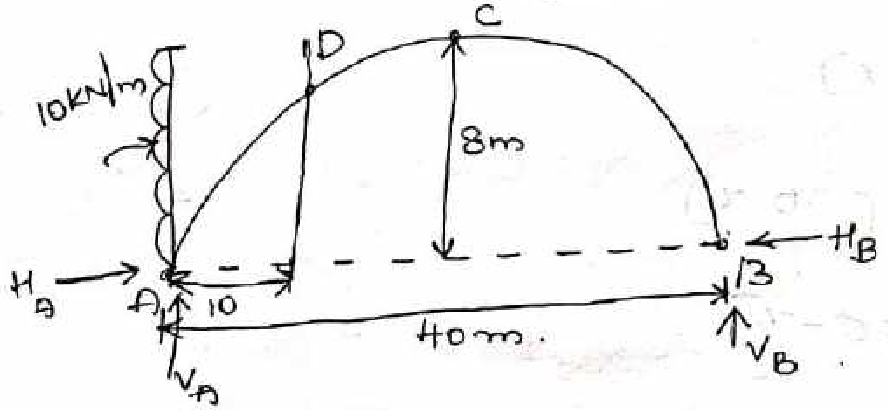
$\theta = 26.56$

Radial shear $= H\sin\theta - V\cos\theta$

$= 500\sin(26.56] - 250\cos[26.56]$

$= -0.049\,kN$ $\underline{-0.049\,kN}$

A 3 hinged parabolic arch of span 40m and central rise 8m hinged at the abutment and crown carries UDL of 10 kN/m as shown in fig. Calculate Bending moment normal thrust and Radial Shear @ left quarter Span.



$\Sigma H = 0$

$H_A - H_B = 0$

$H_A - H_B + 10 \times 8 = 0$ $\quad H_A - H_B = -80$

$\Sigma V = 0$

$V_A + V_B = 0$

$V_A = -V_B$

$\Sigma M_A = 0$

$(H_A \times 0) + (H_A \times 0) - V_B \times 40 + (10 \times 8)\, 8/2 = 0$

$-40 V_B + 320 = 0$

$320 = 40 V_B$

$\boxed{V_B = 8\,kN}$

$V_A = -V_B$

$\boxed{V_A = -8\,KN}$.

$\Sigma M_B = 0$

$-H_A \times 8 + (V_A \times 20) - (10 \times 8)\, 4$

$-H_A \times 8\ (-8 \times 20) - 320 = 0$

~~$H_A \times 8 = 160$~~

~~$H_A = 10$~~

$8H_A = -320 - 160$

$8H_A = -480$

$\boxed{H_A = -60}$

$\Sigma M_C = 0$

$(H_B \times 8) + V_B \times$

$H_B = H_A + 80$

$= -60 + 80$

$\boxed{H_B = 80\,KN}$

$$y = \frac{4hx}{l^2}(l - x]$$

$$y = \frac{4(8)x}{40^2}[40 - x]$$

$$y = 0.02x(40 - x].$$

$$y = 0.8x - 0.02x^2 \quad x = 30 \quad y = 6$$

$$\tan\theta = \frac{dy}{dx}$$

$$= \frac{d}{dx}[0.8x - 0.02x^2]$$

$$\tan\theta = 0.8 - 0.04x$$

$$\theta = \tan^{-1}[0.8 - 0.04(10)]$$

$$\theta = 30.96 \quad \boxed{\theta = 21.80}$$

$$BMM = (V_B \times 30) - [H_B \times 8] = 0$$

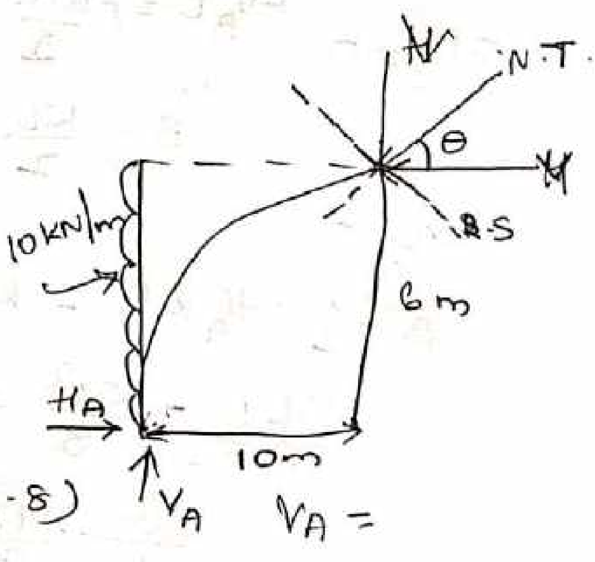
$$= (8 \times 30) - [20 \times 8]. \; [240 - [20[6]]$$

$$BM_D = 120 \text{ kN-mm}$$

$$BMM_D = (V_A \times 10) - (H_A \times 6) - (10 \times 6)(6/2]$$

$$= [(-8)(10)] - [(20)(6)] - [10 \times 6](3)$$

$$\boxed{BM_D = 100 \text{ kN-m}}$$



Normal thrust

$$H_A + (10 \times 6) = 0.$$

$$H = -60 + 60 = 0$$

$$N.T = H\cos\theta + V\sin\theta$$

$$= 0(\cos 21.8) + (-8)\sin(21.8)$$

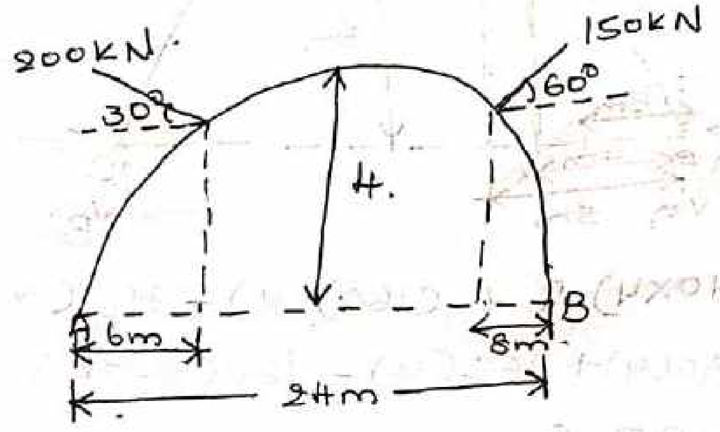
$$= -2.970 \text{ kN}$$

$$R.S = H\sin - V\cos\theta$$

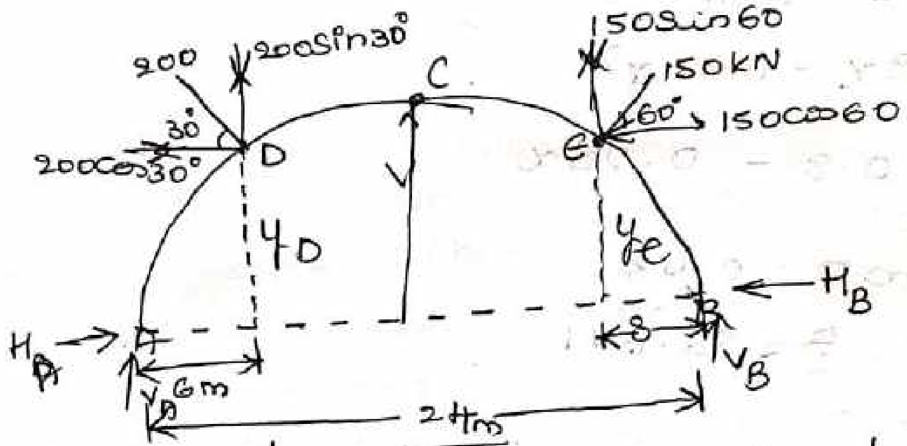
$$= -(-8)\cos(21.8)$$

$$R.S = 7.427 \text{ kN}$$

A 3 hinged parabolic arch is loaded as shown in a fig. determine the bending moment @ loaded points.



Soln:



Note: Donot take horizontal resolved force. Consider

$$\Sigma H = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B$$

$$\Sigma V = 0$$

$$V_A + V_B - 200 \sin 30 - 150 \sin 60° = 0$$

$$V_A + V_B = +229.90 \, KN$$

$$\Sigma M_A = 0$$

$$+200 \sin 30 (6) + 150 \sin 60° \times 16 - V_B \times 24 = 0$$

$$V_B \times 24 = 2678.46$$

$$\boxed{V_B = 111.60 \, KN}$$

$$V_A = 229.9 - 111.6$$

$$\boxed{V_A = 118.3 \, KN}$$

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$\Sigma M_C = 150 \sin 60^\circ (4) - H_B \times h$

$\Sigma M_A = 0$

$200 \sin 30 (6) + 150 \sin 60^\circ (16) - V_B \times 24$
$- [200 \cos 30 \times y_D] - [150 \cos 60^\circ \times y_E] = 0$

$\boxed{V_B = 122.15 \text{ kN}}$ $\boxed{V_A = 107.75 \text{ kN}}$

$y = \frac{4hx}{l^2}(l - x)$

$= \frac{4(4)x}{24^2}(24 - x)$

$= 0.66x(24 - x)$

$= 16x - 0.66x^2$

$\Sigma M_C = 0$

$-(H_A \times 4) + (V_A \times 12) - 200 \cos 30 (1) y_D = 16(6) - 0.666^2$

$- 200 \sin 30 (6) = 0$

$y = \frac{4 \times 4 \times 6}{24^2}(24 - 6)$

$H_A \times 4 = 107.75(12) - 200 \cos 30^\circ - 200 \sin 30 (6)$

$\boxed{y_D = 3 \text{ m}}$

$\boxed{H_A = \cancel{107.40}\ 129.94 \text{ kN}}$ (↑ +ve)

$y_E = \frac{4(4)(8)(24-8)}{24^2}$

$\boxed{y_E = 3.55 \text{ m}}$

$\Sigma M_C = 0$

$(V_B \times 12) - (H_B \times 4) - 150 \cos 60 (1.5) - 150 \sin 60^\circ (4)$

$\boxed{H_B = 129.94 \text{ kN}}$

$BM_D = -(129.94)(3) + 107.75(6)$
$= 259.5 \text{ kN-m}$

$BM_E = + V_B \times 18 - H_B (3.5)$
$= 122.15(8) - 129.94(3.5)$

$\boxed{BM_E = 522.41 \text{ kN-m}}$

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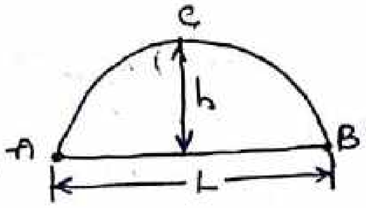
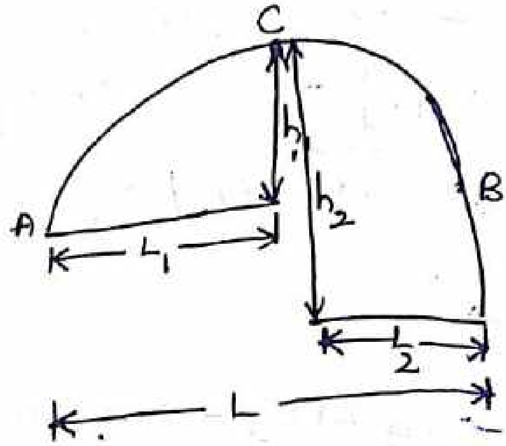
3 Hinged Arch parabolic arch with Supports at Different levels

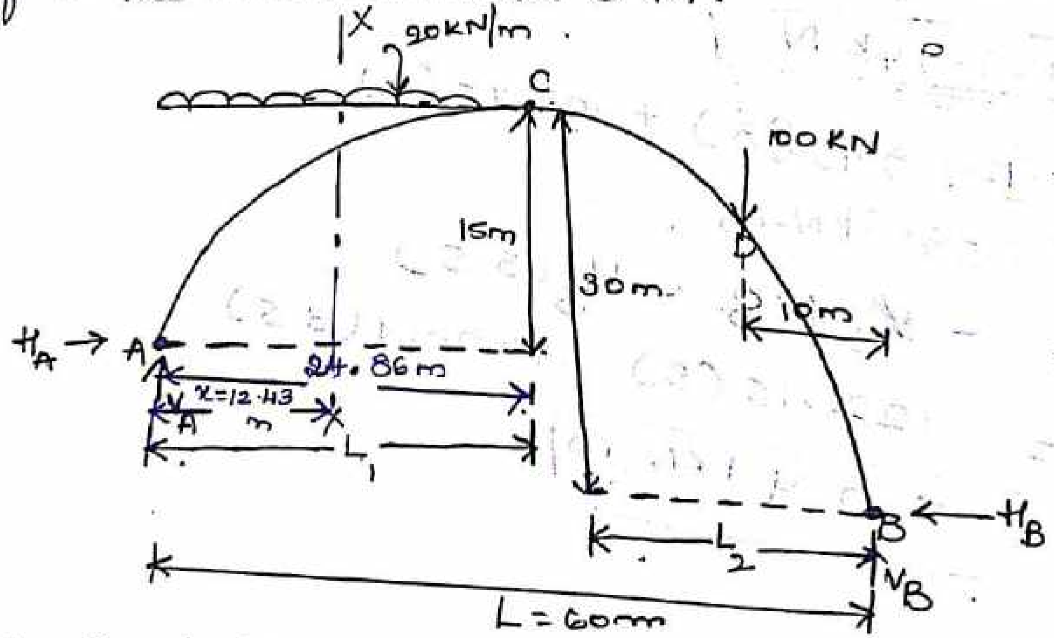




problem

1. A Three hinged arch having supports at different levels of span 60m its abutment A & B are of 15m and 30m from crown C. The arch carries UDL of 20kN over the portion AC and point of load of 100kN at a point 10m from B. Find the reaction Normal thrust radial shear and Bending moment at 15m from A. Also find the maximum B.M.

Soln:



Step 1 : Find Horizontal Distance L_1 & L_2

$L = L_1 + L_2 = 60$

W.K.T

$L = L_1 + L_2 = 60$

$\frac{L_1}{L} = \sqrt{\frac{h_1}{h}}$

$$\frac{L_1}{L_2} = \sqrt{\frac{15}{30}}$$

$$\frac{L_1}{L_2} = 0.707$$

$$\boxed{L_1 = 0.707 L_2}$$

$$\therefore L_1 + L_2 = 60$$

$$0.707 L_2 + L_2 = 60$$

$$1.707 L_2 = 60$$

$$L_2 = \frac{60}{1.707}$$

$$\boxed{L_2 = 35.14 m}$$

$$\therefore L_1 = 60 - L_2$$

$$L_1 = 60 - 35.14$$

$$\boxed{L_1 = 24.86 m}$$

step 2: To find Reaction

$$\Sigma M_A = 0$$

$$(H_A \times 0) + (V_A \times 0) - (V_B \times 60) + (H_B \times 15) + (100 \times 50) + (20 \times 24.86)\left[\frac{24.86}{2}\right] = 0$$

$$-60 V_B + 15 H_B + 11180.196 = 0$$

$$-60 V_B + 15 H_B = -11180.196 \rightarrow 1$$

$$\Sigma V = 0$$

$$V_A + V_B - 100 - (20 \times 24.86) = 0$$

$$V_A + V_B = 100 + (20 \times 24.86)$$

$$V_A + V_B = 597.2$$

$\Sigma M_C = 0$ [Take portion BC]

$$-(V_B \times 35.14) + (H_B \times 30) + 100 \times (35.14 - 10) = 0$$

$$-35.14 V_B + 30 H_B = -2514 \rightarrow 2$$

Solving eqn ① & ② we get

$$-60V_B + 15H_B = -11180.196 \quad \times (2)$$
$$-35.14V_B + 30H_B = -2514 \quad \times (1)$$

$$-120V_B + 30H_B = -22360.39$$
$$\underline{\underset{(+)}{-}35.14V_B \underset{(-)}{+} 30H_B = \underset{(+)}{-}2514}$$
$$-84.86V_B = -19846.39$$

$$\therefore -84.86V_B = -19846.39$$

$$V_B = \frac{-19046.39}{-84.86}$$

$$\boxed{V_B = 233.872 \text{ KN}}$$

$$V_A + V_B = 597.2$$

$$\therefore V_A = 597.2 - 233.872$$

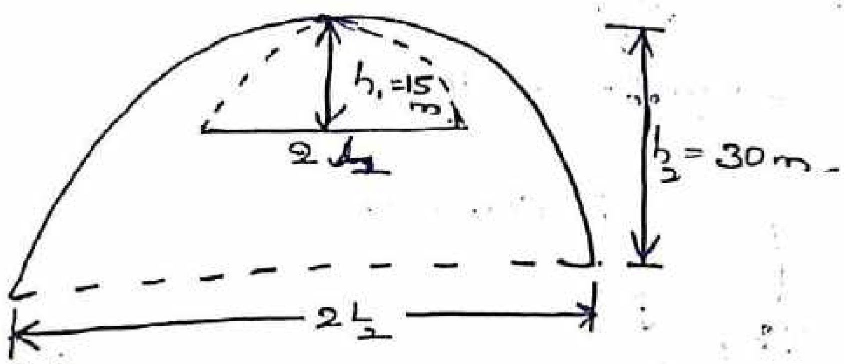
$$\boxed{V_A = 363.32 \text{ KN}}$$

$$-60V_B + 15H_B = -11180.196$$
$$-60(233.876) + 15H_B = -11180.196$$
$$15H_B = -11180.196 + 60(233.876)$$
$$15H_B = 2852.364$$
$$H_B = \frac{2852.364}{15}$$
$$\boxed{H_B = 190.13} \text{ KN}$$

Step3 : Compute the Vertical ordinate y and slope θ For length L

W.K.T

$$y = \frac{4hx}{l^2}[l - x]$$



for length L_1 :→

$$\therefore\ y = \frac{4hx}{L^2}[L-x]$$

$$y = \frac{4(15)x}{(2L_1)^2}[2L_1 - x] \Rightarrow \frac{4(15)x}{[2(24.86)]^2}[2(24.86)-x]$$

$$\boxed{y = 1.206x - 0.024x^2}$$

$$\frac{dy}{dx} = \tan\theta$$

$$\tan\theta = \frac{d}{dx}[1.206x - 0.024x^2]$$

$$\boxed{\tan\theta = 1.206 - 0.048x}$$

for length L_2 :→

$$y = \frac{4h_2x}{(2L_2)^2}(2L_2 - x]$$

$$y = \frac{4(30)x}{[2(35.14)]^2}[2(35.14)-x]$$

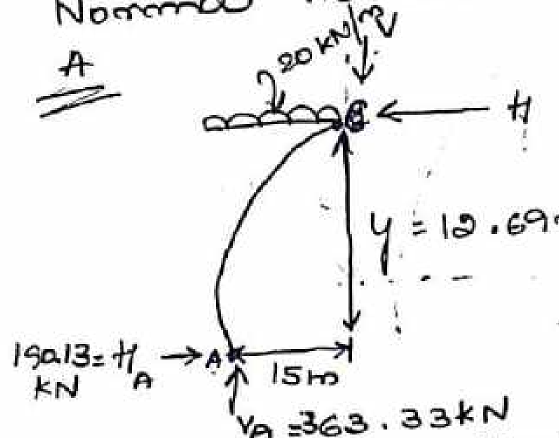
$$\boxed{y_2 = 1.707x - 0.024x^2}$$

$$\therefore\ \tan\theta = \frac{d}{dx}[1.707x - 0.024x^2]$$

$$\boxed{\tan\theta = 1.707 - 0.048x}$$

Step-4 : Normal thrust and Radial Shear @ 15m fr.

A



$$\boxed{H = H_A = 190.13\,kN}$$

$V = V_A - (20 \times 15)$

$= 363.33 - (20 \times 15)$

$$\boxed{V = 63.33\,kN}$$

@ $x = 15m$

$y = (1.206 \times x) - (0.024 \times x^2)$

$= 1.206(15) - 0.024(15)^2$

$$\boxed{y = 12.69m}$$

$\tan\theta = 1.206 - 0.048x$ @ $x = 15$

$= 1.206 - 0.048(15)$

$\tan\theta = 0.486$

$\theta = \tan^{-1}(0.486)$

$$\boxed{\theta = 25^\circ 97'}$$

Normal Thrust (N) $= H\cos\theta + V\sin\theta$

$= 190.13\cos(25^\circ 97') + (63.33)\sin(25^\circ 97')$

$$\boxed{N = 198.66\,kN}$$

Radial Shear $= H\sin\theta - V\cos\theta$

$= 190.13\cos(25^\circ 97') - (63.33)\cos(25^\circ 97')$

$= 28.563\,kN$

Step 5 : Calculation of Bending moment @ 15m from A

$$BM @ 15m = M = V_A \times 15 - (20 \times 15 \times 15/2) - (H_A \times y)$$

$\boxed{y = 12.69m}$ from ordinate eqn

$$M = (363.33 \times 15) - [20 \times 15 \times 15/2] - (H_A \times 12.69)$$

$$M = (363.33 \times 15) - (20 \times 15 \times 15/2] - [190.13 \times 12.69]$$

$$\boxed{M = 787.20 \, KN\text{-}m}$$

Step-6 : Calculation of maximum BM.

Since UDL is for complete left portion. maximum Bending moment can be taken exactly @ midpoint of AC

i.e $x = \frac{L_1}{2} = \frac{24.86}{2}$

$x = 12.43m$

$\therefore$ @ $x = 12.43m$

$$y = (1.204x) - 0.024x^2$$

$$= 1.204(12.43) - 0.024(12.43)^2$$

$$y = 11.29m$$

$$BM_{xx} = (V_A \times 12.43) - (20 \times 12.43 \times 12.43/2] - (H_A \times y)$$

$$= [363.33(12.43)] - [20 \times 12.43 \times 12.43/2] - [190.13 \times 11.29]$$

Max $\boxed{BM_{xx} = 824.5 \, KN\text{-}m}$

BM @ Load point.

$$M_D = (V_B \times 10) - (H_B \times y)$$

$$y = 1.707x - 0.024x^2 \quad @ \; x = 10$$

$$y = 1.707(10) - 0.024(10)^2$$

$$\boxed{y = 14.67m}$$

$\therefore M_D = (233.87 \times 10) - (190.13 \times 14.67)$

$$\boxed{M_D = -450.50 \text{ KN-m}}$$

$\therefore$ Max^m +ve BM $= 824.5$ KN-m

Max^m -ve BM $= -450$ KN-m

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